Game: Draw one card from a standard poker deck.

Rules: If you draw an Ace, you win \$5
If you draw a face card (Jack, Queen, King), you lose \$5
If you draw any other card (2-10), you neither win nor lose.

Question: If you play this game 100 times, what is the probability that your total winnings are nonnegative.

Solution: Let W_i = winnings on ith play.

$$W_i = \begin{cases} +5 & \text{if } A \\ 0 & \text{if } 2\text{-}10 \\ -5 & \text{if } J, Q, K \end{cases} \qquad \begin{aligned} P(W_i = 5) &= 1/13 \\ P(W_i = 0) &= 9/13 \\ P(W_i = -5) &= 3/13 \end{aligned}$$

$$\mu = E(W_i) = (5)\left(\tfrac{1}{13}\right) + (0)\left(\tfrac{9}{13}\right) + (-5)\left(\tfrac{3}{13}\right) = -\tfrac{10}{13} \approx -0.77$$

$$E(W_i^2) = (25)\left(\tfrac{1}{13}\right) + (0)\left(\tfrac{9}{13}\right) + (25)\left(\tfrac{3}{13}\right) = \tfrac{100}{13}$$

$$\sigma^2 = Var(W_i) = \left(\tfrac{100}{13}\right) - \left(-\tfrac{10}{13}\right)^2 = \tfrac{1200}{169} \approx 7.1 = (2.66)^2$$

Let $S_{100} = W_1 + \cdots + W_{100}$ = total winnings $(n = 100)$

By the Central Limit Theorem,

$$S_{100} \approx N(n\mu, n\sigma^2) = N(-77, 710) = N(-77, (26.6)^2)$$

$$P(S_{100} \geq 0) \approx P\left(Z \geq \frac{0-(-77)}{26.6}\right) = P\left(Z \geq \frac{77}{26.6}\right)$$

$$= P(Z \geq 2.89) = 1 - \underbrace{P(Z < 2.89)}_{0.9981}$$

$$= 1 - 0.9981 = \boxed{0.0019}$$

Conclusion: If you play this game 100 times, your chances of not losing money (overall) are 0.19%.

Real conclusion: Don't play this game.

Comment: In class, I used $\sigma = 7.1$. (I forgot the square root.)