Directions: These are examples of the types of questions that will be on the exam. The actual exam will consist of 5 questions, each one similar in style to one of the problems below. You do not need to evaluate factorials such as 4! or expressions like $\binom{5}{3}$. The more work you show and more explanation you give, the more partial credit we will be able to assign.

1. Ten children are seated in a row. How many different seating arrangements are possible if Alan refuses to sit next to Beth or Carl?

2. Chris and his sister Meg are in a dance class that consists of 14 boys and 16 girls. A team of five boys and five girls will be selected to compete in a dancing competition. If each of the five boys is to be randomly paired with one of the five girls for the competition, what is the probability that Chris will have to dance with his sister?

3. "Ah, Mr. Bond!" said Ernst Stavro Blofeld as he stroked his pet cat. "I will tell you my cunning plan for world domination if you can randomly draw five cards from this fair deck all of the same suit." Bloefeld gave a small chuckle. "But if you cannot," his eyes narrowing, "I will introduce you to my carnivorous gold fish." What is the probability that Bond will survive this encounter? (The ranks are: Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3, 2, and there are four suits of each rank in a standard deck.)

4. John walks into a cheese shop looking to buy some cheese. The shop has cheeses from France and Belgium. Two-thirds of the cheeses are French, while the rest are Belgian. Suppose 10% of the French cheeses are pasteurized, while 20% of the Belgian cheeses are pasteurized. If John randomly tastes a cheese and it turns out to be pasteurized, what is the probability this cheese started life in Belgium?

5. A single card is drawn from a standard deck of 52 cards, and the rank is noted. The card is replaced, the deck randomly shuffled, and the process is repeated until a face card appears (that is, a Jack, Queen, or King). What is the probability an Ace is drawn before a face card?

6. On a given day, 80 birds fly over your car. If each bird has a 10% chance of making a mess on your car, then what is the probability you will need to wash your car at the end of the day? How many birds do you expect will make a mess on your car?

7. Suppose you are playing a game where you draw one card out of a standard deck. If the card is an ace, king, queen, or jack, you win \$10 (probability $\frac{4}{13}$). If the card is anything else, you lose \$5. If you win the first two, you stop. Otherwise, you place a third bet, and then stop. Should you play this game? (Hint: Write out all of the possible outcomes.)

8. A continuous random variable X has a probability density function given by the formula

$$f(x) = \begin{cases} cx(1-x) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find c, $E(X)$, $\text{Var}(X)$, and the cumulative distribution function $F(u)$.