

Lectures in Differential Calculus

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## Preface

This book is based on lecture notes used in the Math 10A calculus course offered at UC San Diego (as taught by me). It was originally intended to accompany the lectures that I recorded for Math 10A during Spring Quarter 2021, while instruction was remote. (Those lectures can be found on my YouTube channel.) While this book did end up growing somewhat beyond its original scope, I did for the most part remain true to the course as it was given. There were 26 lectures given, and there are 26 lectures included in this book. In an ideal world, some of the lectures (especially the sections on extreme values and optimization) would be given more time, but I was constrained by the number of days available for lecture during that quarter.

At the end of each section, there are exactly ten exercises. I tried to offer exercises that are straightforward, but also interesting. Some are more challenging than others, and I hope I gave enough hints to keep them from becoming intractable. You will also find a number of exercises taken from the sciences. The purpose of these exercises is to show that calculus does actually show up in practical applications. Please don't be put off by all of the words. Problems taken from the sciences require some amount of setup in order to understand the context, but if you can set aside all of that, the problems themselves are (I hope!) straightforward enough. You don't usually need the context in order to solve the problems, but I believe the context gives some extra life to the problems, so they don't seem abstract and esoteric.

There are many calculus books out there—far too many to count, in fact. Does the world need another? Probably not. I wrote this book because I wanted to write it. I wanted to be able to hold something in my hand that was (for me) a result of all the work I put into teaching calculus during the infamous year of 2021. I don't know if anyone is going to read this far, but if you do, I hope you find something in this book that you think is interesting. Calculus is a fascinating subject, and one that still teaches me new things even after all these years of teaching it to others.