

Midterm MAT 214: Theorems and Problem Sets.

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$3\frac{1}{2}$ -hour exam

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1-(20 points) Show that a real number has a periodic simple continued fraction if and only if it is an irrational quadratic number.

2-(15 points) State and prove lemma of Gauss for proving quadratic reciprocity.

3-(15 points) Show that any prime number of the form $3k + 1$ can be written as $x^2 + xy + y^2$ for some integers x and y .

4-(5 points) Given $\sqrt{18} = \langle 4, \overline{4, 8} \rangle$, find the least positive solution of $x^2 - 18y^2 = -1$ if any and of $x^2 - 18y^2 = 1$.

5-(5 points) Show that, for all positive integers m , $\sum_{m|n} d(m)^3 = (\sum_{m|n} d(m))^2$, where $d(m)$ is the number of positive divisors of m .