

**MATH 109: THE FIRST EXAM.**  
**INSTRUCTOR: A. SALEHI GOLSEFIDY**

NAME: .....

PID: .....

- (1) Write your Name and PID on the front of your exam sheet.
- (2) No calculators or other electronic devices are allowed during this exam.
- (3) Show all of your work; no credit will be given for unsupported answers.
- (4) Read each question carefully to avoid spending your time on something that you are not supposed to (re)prove.
- (5) Ask me when you are unsure if you are allowed to use certain fact or not.

| Problem         | Score out of 10 |
|-----------------|-----------------|
| 1               |                 |
| 2               |                 |
| 3               |                 |
| Total out of 30 |                 |

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*Date: 04/29/2013.*

(1) Which one of the following propositional forms is NOT equivalent to  $P \Rightarrow (Q \vee R)$ ? Justify your answer.

(a)  $(P \wedge (\neg Q) \wedge (\neg R)) \Rightarrow O$ .

(b)  $(P \wedge (\neg Q)) \Rightarrow R$ .

(c)  $(P \Rightarrow Q) \wedge (P \Rightarrow R)$ .

(d)  $(\neg P) \vee Q \vee R$ .

(You have to only prove why your chosen propositional form is not equivalent to  $P \Rightarrow (Q \vee R)$ . You do NOT need to argue why the rest are equivalent.)

- (2) Prove that there are no integers  $m$  and  $n$  such that  $7m + 21n = 15$ .

(3) Let  $a_1 = 1$  and

$$a_{n+1} = 1 + \frac{1}{1 + \frac{1}{a_n}}$$

for any positive integer  $n$ . Prove that for any positive integer  $n$  we have that  $a_n < a_{n+1}$ . (Hint: if  $0 < b < c$ , then  $\frac{1}{c} < \frac{1}{b}$ . And you can assume  $a_n > 0$  without proof.)