

MIDTERM MATH 200B.

NAME: .....

1. (a) (2 points) Define a left semisimple ring.

(b) (5 points) Let  $R$  be a left semisimple ring. Prove that there are simple  $R$ -modules  $I_1, \dots, I_k$  and positive integers  $n_1, \dots, n_k$  such that

- (1)  $I_i \not\simeq I_j$  if  $i \neq j$ .
- (2)  $R \simeq \bigoplus_{i=1}^k I_i^{n_i}$  as  $R$ -modules.

(c) (5 points) Let  $I$  and  $J$  be two simple  $R$ -modules. Prove that if  $f \in \text{Hom}_R(I, J)$  is not zero, then it is an isomorphism. Then conclude that

- (1)  $\text{Hom}_R(I, J) \neq 0$  if and only if  $I \simeq J$  as  $R$ -modules.
- (2)  $\text{End}_R(I)$  is a division ring.

(d) (10 points) Let  $R$  be a left semisimple ring and  $I_i$  and  $n_i$  as in part (b). Prove that

$$R \simeq \bigoplus_{i=1}^k \text{End}_R(I_i^{n_i}).$$

(Hint: Consider  $\text{End}_R(R)$ ).

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2. (a) (10 points) Let  $A \in M_n(\mathbb{C})$ . Prove that  $N$  is nilpotent if and only if  $N^n = 0$ .

(b) (5 points) Give a list of  $5 \times 5$  matrices such that any nilpotent matrix in  $M_5(\mathbb{C})$  is similar to one and only one of them.

(c) (5 points) Give the minimal polynomials of each one of the matrices in part (b). (You do not have to justify your answer!)

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3. Let  $A \in M_n(\mathbb{C}[x])$ . Assume that  $\det(A) \neq 0$ .

(a) (10 points) Prove that there are complex polynomials  $q_1(x)|q_2(x)|\dots|q_n(x)$  and  $P_1, P_2 \in \text{GL}_n(\mathbb{C}[x])$  such that

- (1)  $\mathbb{C}[x]^n/\text{Im}(A) \simeq \bigoplus_{i=1}^n \mathbb{C}[x]/q_i(x)$  as  $\mathbb{C}[x]$ -modules.
- (2)  $P_1 A P_2 = \text{diag}(q_1(x), \dots, q_n(x))$  i.e. it is a diagonal matrix with diagonal entries equal to  $q_i(x)$ .

(Hint: Consider  $\text{Im}(A) \subseteq \mathbb{C}[x]^n$ .)

(b) (10 points) Prove that  $\dim_{\mathbb{C}}(\mathbb{C}[x]^n/\text{Im}(A)) = \deg(\det(A))$ . (Hint: Use part (a)!)

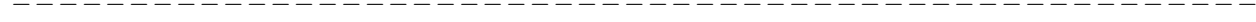
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GOOD LUCK!

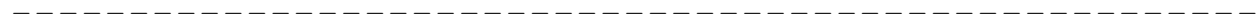
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Date: February 11, 2013.

1. (a)

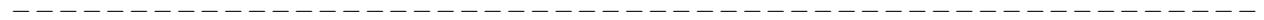


1. (b)

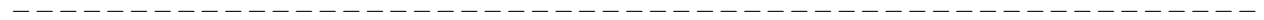


1. (c)

1. (c)(1)



1. (c)(2)



1. (d)

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NAME: .....

2. (a)

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2. (b), (c)

3. (a)

3. (b)