Final of *Numbers, equations, and proofs*
Theorems and problem sets.

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Jan 13, 2009

Hi! This is a 6-hour exam. This means as soon as you open this file your exam starts, and it finishes after six hours. For this exam, you are allowed to look at your notes and the course book. However you are not allowed to use any other book, internet, etc. You are not allowed to talk about the exam to anyone, till you hand in your exam. You will hand in your exam by Friday, Jan 16, 7:00 pm. I will be in my office most of Friday afternoon. Good luck!

1-a) (5 points) Show that for any x, y real numbers

$$\lfloor 2x \rfloor + \lfloor 2y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor + \lfloor x+y \rfloor.$$

b) (5 points) Let m and n be positive integer numbers. Show that

$$\frac{(2m)!(2n)!}{m!n!(m+n)!}$$

is an integer number.

2- (10 points) Show that $(2^n-1, 2^m-1) = 2^{(n,m)}-1$ for any pair of positive integers n and m.

3- (10 points) Assume that any integer number appears once and only once in either $\{a_n\}$ and $\{b_n\}$. Furthermore assume that $a_n - b_n = n$ for any n. Find a_n and b_n. (*Hint: Use Beaty's sequence.*)

4- (10 points) Let $f : \mathbb{N} \to \mathbb{N}$ be a function from positive integers to positive integers such that

$$2^n = \sum_{d|n} f(d),$$

for any $n \geq 1$. Show that n divides $f(n)$.

5- (10 points) Let $\xi = \langle a_0, a_1, a_2, \cdots \rangle$ and c be a real number larger than 2. If $|\xi - \frac{a}{b}| < \frac{1}{b^c}$ for infinitely many rational numbers a/b, then $\{a_n\}$ is unbounded, i.e. there is no M such that $a_n < M$ for any n.