PRACTICE EXAM FOR THE SECOND MIDTERM.

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The exam will be considered out 50.

1. (5 points) Find all the maximal ideals of $\mathbb{Z}/10\mathbb{Z}$.

2.

(1) (5 points) Prove that $x^2 + x + 2$ is irreducible over $\mathbb{Z}/5\mathbb{Z}$.
(2) (5 points) Prove that $F = (\mathbb{Z}/5\mathbb{Z})[x]/\langle x^2 + x + 2\rangle$ is a field of order 25.
(3) (5 points) Find the multiplicative inverse of $2x + 3 + \langle x^2 + x + 2\rangle$ in F.

3. (10 points) Is $(5/2)x^5 + (9/2)x^4 + 15x^3 + (3/7)x^2 + 6x + 3/14$ irreducible over $\mathbb{Q}$?

4. (10 points) Prove that for any integers m and n, the polynomial $x^3 + (5m + 1)x + (5n + 1)$ is irreducible over $\mathbb{Z}$.

5. (20 points) Let F be a field and $f(x), g(x)$ be two non-zero polynomials in $F[x]$. Then there are $p(x), q(x) \in F[x]$ such that

$$\gcd(f(x), g(x)) = p(x)f(x) + q(x)g(x).$$

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