1.

P	Q	R	$Q\vee R$	$P\wedge Q$	$P\wedge R$	$P\wedge(Q\vee R)$	$(P\wedge Q)\vee(P\wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F
						I	II

P	Q	$\neg P$	$\neg Q$	$P\wedge Q$	$\neg(P\wedge Q)$	$(\neg P)\vee(\neg Q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T
					III	IV

P	Q	$\neg P$	$P\Rightarrow Q$	$\neg P\vee Q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T
			V	VI

2. I and II imply that $P\wedge(Q\vee R)\equiv(P\wedge Q)\vee(P\wedge R)$.

III and IV imply that $\neg(P\wedge Q)\equiv(\neg P)\vee(\neg Q)$.

V and VI imply that $P\Rightarrow Q\equiv(\neg P)\vee Q$. VII

$$(\neg P)\vee Q\equiv Q\vee(\neg P)\equiv(\neg Q)\Rightarrow(\neg P)$$

We use VII to deduce the above equivalency.

3. This is NOT a proposition as we cannot assign a truth-value to it.

- If it is true, then it says it is false which is a contradiction.
- If it is false, then it is NOT false which is a contradiction.

4.

x	y	$x<y$	$x=y$	$x \le y$
1	1	F	T	T
1	0	F	F	F
0	1	T	F	T

5.

$x \ge 0$	$y \ge 0$	$x+y \ge 0$	$\lvert x\rvert$	$\lvert y\rvert$	$\lvert x+y\rvert$	$\lvert x\rvert+\lvert y\rvert-\lvert x+y\rvert$	$\lvert x\rvert+\lvert y\rvert \ge \lvert x+y\rvert$
T	T	T	x	y	$x+y$	0	T
T	T	F	x	y	$-x-y$	$-2x-2y$	F
T	F	T	x	$-y$	$x+y$	$-2y$	T
T	F	F	x	$-y$	$-x-y$	$-2x$	T
F	T	T	$-x$	y	$x+y$	$-2x$	T
F	T	F	$-x$	y	$-x-y$	$2y$	T
F	F	T	$-x$	$-y$	$x+y$	$-2x-2y$	F
F	F	F	$-x$	$-y$	$-x-y$	0	T

To get the last column, we look at the value of

$|x|+|y|-|x+y|$ and find out if it is non-negative.

For instance, when it is $\underline{0}$, it is non-negative.

The third entry: it is $\underline{\underline{-2y}}$. In that row,

the truth-value of $\underline{y \geq 0}$ is false. So $y<0$.

Hence $-2y>0$.

In order to show $|x|+|y| \geq |x+y|$, it is enough to say why the second and the seventh cannot happen.

<u>Second row</u>: $x \geq 0$ and $y \geq 0$ imply $x+y \geq 0$.

So the second row cannot happen.

<u>Seventh row</u>: $x<0$ and $y<0$ imply $x+y<0$.

Hence the seventh row cannot happen.

6.

P	Q	R	f(P,Q,R)
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

$f(P,Q,R)$ is true $\Longleftrightarrow$

either $P \wedge Q \wedge (\neg R)$ is true

or $P \wedge (\neg Q) \wedge (\neg R)$ is true

or $(\neg P) \wedge Q \wedge (\neg R)$ is true

Hence $f(P,Q,R)$ is true $\iff$

$$(P\wedge Q\wedge(\neg R))\vee(P\wedge(\neg Q)\wedge(\neg R))\vee((\neg P)\wedge Q\wedge(\neg R))$$

is true.

Therefore

$$f(P,Q,R)\equiv(P\wedge Q\wedge(\neg R))\vee(P\wedge(\neg Q)\wedge(\neg R))\vee((\neg P)\wedge Q\wedge(\neg R))$$

[Students might give a different propositional form (which is equivalent to what I have written here). Of course, it is OK. The advantage of the above solution is the fact that it is algorithmic and shows that any propositional form is equivalent to a propositional form involving only $\wedge, \vee, \neg$ and (,).]

After simplyfing, one can show that

$$f(P,Q,R)\equiv(\neg R)\wedge(P\vee Q)$$

[5. (Alternative)

① $xy\le|xy| \Rightarrow$ ② $x^2+2xy+y^2\le|x|^2+2|xy|+|y|^2$

$$\Rightarrow (x+y)^2\le(|x|+|y|)^2$$

$$\overset{②}{\Longrightarrow} |x+y|^2 \le (|x|+|y|)^2$$

$$\overset{③}{\Longrightarrow} |x+y| \le |x|+|y|.$$

(Clearly this proof is constructed backwards.)

In the above argument we are using the following facts:

① $a \le |a|$ for any real number a.

Pf of ① If $a \ge 0$, then $|a| = a$.

If $a < 0$, then $|a| = -a > 0 > a$. ∎

② $a^2 = |a|^2$ for any real number a.

Pf of ② If $a \ge 0$, then $|a| = a \Rightarrow |a|^2 = a^2$.

If $a < 0$, then $|a| = -a \Rightarrow |a|^2 = a^2$. ∎

③ If a and b are non-negative real numbers and $a^2 \le b^2$, then $a \le b$.

Pf of ③ If not, $a > b \ge 0$. Thus $a^2 > ab \ge b^2$, which is a contradiction. ∎]

[5. (Alternative)

$|a| = a$ if $a \ge 0$ and $-a$ if $a < 0$.

$$\Rightarrow \left\{\begin{array}{l} \pm x \leq |x| \\ \pm y \leq |y| \end{array}\right\} \Rightarrow \pm(x+y) \leq |x| + |y|$$

$$\Rightarrow |x+y| \leq |x| + |y| \text{ . }]$$