Reading Course: Introduction to commutative algebra.

Instructor: Alireza Salehi Golsefidy, Fine 510, 8-4230. Meeting Hours: 10:30 am-12:30 pm Wed (Fine 510) E-mail: asalehi@math.princeton.edu Homework assignments will be available on-line.

1 General information

Nowadays commutative algebra is one of the essential parts of studying mathematics. A solid background in commutative algebra can help a student in his or her study, not only in this subject itself, but also in various other topics such as homological algebra, algebraic geometry, algebraic number theory, non-commutative algebra and algebraic combinatorics.

2 Books

In this course, we will go through the excellent book by M. F. Atiyah and I. G. Macdonald, *Introduction to commutative algebra*. For some topics, we may also use some other resources, e.g. Matsumura's *Commutative algebra*, Sharp's *Steps in commutative algebra* and Ash's *A course in commutative algebra*.

3 Grading

The grading breakdown for this reading course is divided as follows:

- 50% Oral presentations.
- 20% Weekly assignments.
- 30% Final exam.

4 Homework

Each week, I will assign a set of homework exercises, which will be due the next meeting. Most of them will be assigned from the textbook.

5 Schedule

1. 10 Feb: Prime ideals, Maximal ideals, Nilradical radical, Jacobson radical, Extension and contraction.

Read: Chapter 1

2. 17 Feb: Modules, Finitely generated modules, Nakayama's lemma, Exact sequences, Tensor product, Flat modules.

Read: Chapter 2

3. 24 Feb: Ring of fractions, Module of fractions and their relations with tensor product, exact sequences, etc. Localization. What a primary decomposition is.

Read: Chapter 3 and to the end of page 51

- **3 Mar:** Primary decompositions uniqueness theorems. *Read: Chapter 4*
- 10 Mar : Integral extensions, Going up and Going down theorems. Read: Chapter 5, to the end of page 64
- 24 Mar: Valuation rings, Noetherian and Artin rings. Read: Chapter 5 (continue), Chapter 6
- 31 Mar: Hilbert's basis theorem, Existence of the primary decomposition for Noetherian rings. *Read: Chapter 7*
- 7 Apr: More on Artin rings, DVR, Fractional ideals. Read: Chapter 8, Chapter 9
- **9. 14 Apr:** Dimension theory, Hilbert polynomial. *Read: Chapter 11, to the end of page 119*
- 10. 21 Apr: Dimension theory of Noetherian rings, Krull's principal ideal theorem, Regular local rings, Transcendental dimension.
 Read: The rest of Chapter 11
- 11. 28 Apr: Homological methods: Ext, Tor. Read: Chapter 7 from Ash's book