Name: ____________________

PID: ____________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

1. Write your Name, PID, and Section on the front of your Blue Book.
2. No calculators or other electronic devices are allowed during this exam.
3. You may not use books or other assistance during this exam.
4. Read each question carefully, and answer each question completely.
5. Write your solutions clearly in your Blue Book
    (a) Carefully indicate the number and letter of each question.
    (b) Present your answers in the same order they appear in the exam.
6. Show all of your work; no credit will be given for unsupported answers.

1. (10 points) Show that $\mathbb{Z}_{12} \times \mathbb{Z}_9$ is not cyclic.

2. (10 points) Let $G = \langle a \rangle$ be a finite group of size n. Show that

$$|\{g \in G |\ o(g) = n\}| = \phi(n).$$

3. Let H be a subgroup of $G = \langle a \rangle$.
    (a) (5 points) Show that $I_H := \{m \in \mathbb{Z} |\ a^m \in H\}$ is a subgroup of $\mathbb{Z}$.
    (b) (5 points) Show that H is cyclic.

4. (10 points) Let G be a finite group, p be a prime, and X be a finite set. Suppose G acts from left on X, and $|G| = p$. Show that for any $x \in X$ either x is a fixed point, i.e. for any $g \in G$ we have $g \cdot x = x$, or the size $|O(x)|$ of the orbit of x is divisible by p.

    Hint: Think about the connection between $O(x)$ and the stabilizer subgroup G_x. And use Lagrange!

5. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 8 & 1 & 2 & 5 & 3 & 4 & 7 \end{pmatrix}$.
    (a) (3 points) Write σ as a product of disjoint cycles.
    (b) (5 points) Find $o(\sigma)$. Justify your answer.
    (c) (2 points) Are there a 3-cycle c and a 5-cycle c' such that $o(cc') = 8$ (not necessarily disjoint)? Justify your answer. *(This part has nothing to do with the first two parts!)*

6. (10 points) (EXTRA CREDIT) Can the following arrangement happen in the 15-puzzle? Justify your answer.

2	1	4	3
6	5	8	9
7	14	10	11
12	13	15	

(Hint:

1. *Think about permutations in S_{16}.*
2. *What is a single slide as a permutation?*
3. *What can you say about the number of slides to get to this arrangement?)*

Good Luck!