QUIZ 2, VERSION A, MATH100B, WINTER 2021

1. Answer the following questions and briefly justify your answers.
   (a) (2 point) Find all primes p such that $x-1$ is a factor of $x^5 - 2x^4 + 3x^3 + 5x^2 + 6$ in $\mathbb{Z}_p$.
   (b) (3 points) True or false. $\mathbb{Z}[x]$ is a PID.

2. (5 points) Determine whether $f(x) := x^5 - 2x^4 + 5x^3 - x + 1$ has a zero in $\mathbb{Q}$. Justify your answer.

3. Recall that $\mathbb{Z}[i] = \{a + bi \mid a, b \in \mathbb{Z}\}$.
   (a) (4 points) Prove that $5 + 2i$ is irreducible in $\mathbb{Z}[i]$.
   (Hint: Think about $N(a + bi) = |a + bi|^2 = a^2 + b^2$.)
   (b) (4 points) Prove that $\mathbb{Z}[i]/\langle 5 + 2i \rangle$ is a field.
   (c) (2 points) Prove that the characteristic of $\mathbb{Z}[i]/\langle 5 + 2i \rangle$ is 29.

4. Suppose E is a field extension of $\mathbb{Z}_3$, and $\alpha \in E$ is a zero of $x^3 - x + 2$.
   (a) (6 points) Prove that $\mathbb{Z}_3[\alpha]$ is a field of order 27.
   (b) (2 points) Prove that $\alpha^{26} = 1$. (Hint: Think about $(\mathbb{Z}_3[\alpha])^\times$.)
   (c) (2 points) Prove that $x^3 - x + 2$ divides $x^{26} - 1$.