QUIZ 2, VERSION B, MATH100B, WINTER 2021

1. (3 points) Suppose I is an ideal of a unital commutative ring A and A/I is a finite integral domain. Show that I is a maximal ideal.

2. (5 points) Suppose D is an integral domain, $f, g \in D[x]$ are polynomials of degree at most n, and $a_1, \ldots, a_{n+1}$ are distinct elements of D. Prove that if $f(a_i) = g(a_i)$ for every i, then $f(x) = g(x)$.

3. (5 points) Determine whether $f(x) := x^{3^{2021}} - x + 100$ has a zero in $\mathbb{Q}$. Justify your answer.

4. Suppose $\alpha \in \mathbb{C}$ is a zero of $x^3 - x + 1$.
   (a) (3 points) Find the minimal polynomial of α over $\mathbb{Q}$.
   (b) (4 points) Argue why $(\alpha^2 + 1)^{-1}$ can be written as $a_0 + a_1\alpha + a_2\alpha^2$ for some $a_i \in \mathbb{Q}$. (You are allowed to use all the results proved in the lectures after carefully stating them.)

5. Suppose D is an integral domain which is not a field and $a \in D$.
   (a) (4 points) Prove that $x - a$ is irreducible in $D[x]$.
   (b) (4 points) Prove that $D[x]/\langle x - a\rangle \simeq D$.
   (c) (2 points) Prove that $D[x]$ is not a PID.