QUIZ 2, VERSION A, MATH103A, SUMMER 2021

1. Determine if the following statements are true or false. Briefly justify your answer.
   (a) (2 point) $(\mathbb{Z} \setminus \{0\}, \cdot)$ is a group.

   (b) (2 point) $\mathbb{Q} \setminus \{0\}$ is a subgroup of $(\mathbb{Q}, +)$.

   (c) (2 points) $(\mathbb{Z}_{15} \setminus \{[0]_{15}\}, \cdot)$ is a group.

   (d) (2 point) $[2]_7$ is in the kernel of f where $f : \mathbb{Z}_7^\times \to \mathbb{Z}_7^\times, f([a]_7) := [a]_7^3$.

   (e) (2 point) S_3 is abelian.

2. (5 points) Explain why $[13]_{20}$ is in $\mathbb{Z}_{20}^\times$ and find an integer x such that $[13]_{20}^{-1} = [x]_{20}$.

3. (5 points) Suppose G is a group and $f : G \to G, f(g) = g^2$ is a group homomorphism. Prove that G is abelian.

4. Suppose n is a positive integer and $n \geq 3$. Let $\mathcal{C}_n$ be the cycle with n vertices which are labeled by elements of $\mathbb{Z}_n$ and $[x]_n$ is connected to $[y]_n$ if and only if $[y]_n - [x]_n = \pm[1]_n$. Recall that the group of symmetries of $\mathcal{C}_n$ is the dihedral group D_{2n} with $2n$ elements and

$$D_{2n} = \{\text{id}, \sigma, \ldots, \sigma^{n-1}, \tau, \tau \circ \sigma, \ldots, \tau \circ \sigma^{n-1}\} \tag{1}$$

   where $\sigma : \mathbb{Z}_n \to \mathbb{Z}_n, \sigma(x) := x + 1$ and $\tau : \mathbb{Z}_n \to \mathbb{Z}_n, \tau(x) := -x$. Let's recall that $\sigma^n = \text{id}$, $\tau^2 = \text{id}$, and $\tau \circ \sigma \circ \tau^{-1} = \sigma^{-1}$.
   (a) (4 points) Find out which element of D_{10} is $\tau \circ \sigma^2 \circ \tau$ (an element of the form given in (1)).

   (b) (4 points) Show that σ^3 is not in the centralizer group $C_{D_{10}}(\tau)$.

   (c) (2 points) Suppose $\gamma \in D_{10}$, $\gamma([0]_{10}) = [4]_{10}$, and $\gamma([1]_{10}) = [3]_{10}$. Find out which element of D_{10} is γ (an element of the form given in (1)).