## QUIZ 2, VERSION B, MATH103A, SUMMER 2021

1. Determine if the following statements are true or false. Briefly justify your answer.
   (a) (2 point) $(\mathbb{Z}^{\geq 0}, +)$ is a group, where $\mathbb{Z}^{\geq 0}$ is the set of non-negative integers.

   (b) (2 point) $\mathbb{R} \setminus \{0\}$ is a subgroup of $(\mathbb{R}, +)$.

   (c) (2 points) $(\mathbb{Z}_{21} \setminus \{[0]_{21}\}, \cdot)$ is a group.

   (d) (2 point) $[5]_{13}$ is in the kernel of $f$ where $f : \mathbb{Z}_{13}^{\times} \to \mathbb{Z}_{13}^{\times}, f([a]_{13}) := [a]_{13}^4$.

   (e) (2 point) $|Z(S_n)| = 1$ if $n \geq 3$.

2. (5 points) Explain why $[8]_{19}$ is in $\mathbb{Z}_{19}^{\times}$ and find an integer $x$ such that $[8]_{19}^{-1} = [x]_{19}$.

3. (5 points) Suppose $G$ is a group and $f : G \to G, f(g) = g^{-1}$ is a group homomorphism. Prove that $G$ is abelian.

4. Suppose $n$ is a positive integer and $n \geq 3$. Let $\mathcal{C}_n$ be the cycle with $n$ vertices which are labeled by elements of $\mathbb{Z}_n$ and $[x]_n$ is connected to $[y]_n$ if and only if $[y]_n - [x]_n = \pm[1]_n$. Recall that the group of symmetries of $\mathcal{C}_n$ is the dihedral group $D_{2n}$ with $2n$ elements and

$$D_{2n} = \{\mathrm{id}, \sigma, \ldots, \sigma^{n-1}, \tau, \tau \circ \sigma, \ldots, \tau \circ \sigma^{n-1}\} \tag{1}$$

   where $\sigma : \mathbb{Z}_n \to \mathbb{Z}_n, \sigma(x) := x + 1$ and $\tau : \mathbb{Z}_n \to \mathbb{Z}_n, \tau(x) := -x$. Let's recall that $\sigma^n = \mathrm{id}$, $\tau^2 = \mathrm{id}$, and $\tau \circ \sigma \circ \tau^{-1} = \sigma^{-1}$.
   (a) (4 points) Find out which element of $D_{14}$ is $\tau \circ \sigma^2 \circ \tau \circ \sigma$ (an element of the form given in (1)).

   (b) (4 points) Show that $\sigma^4$ is not in the centralizer group $C_{D_{14}}(\tau \circ \sigma)$.

   (c) (2 points) Suppose $\gamma \in D_{14}$, $\gamma([0]_{14}) = [6]_{14}$, and $\gamma([1]_{14}) = [5]_{14}$. Find out which element of $D_{10}$ is $\gamma$ (an element of the form given in (1)).