Homework 10 (not due)

Friday, June 8, 2018 10:58 PM

1. Suppose E is a finite field. Prove $\prod_{\substack{\alpha\in E\\ \alpha\neq 0}} \alpha = (-1)^{|E|}$

(Hint. Suppose $|E|=q$. Use $x^q - x = \prod_{\alpha\in E}(x-\alpha)$.)

2. Suppose p is prime, $n\in\mathbb{Z}^+$, $p\nmid n$, and E is a field of characteristic p. Prove that x^n-1 does not have a zero with multiplicity more than 1.

3. Suppose $f(x)\in\mathbb{Z}_p[x]$ is irreducible of degree n. Prove that $f(x) \mid x^{p^n}-x$.

(Hint. Let $E:=\mathbb{Z}_p[x]/\langle f(x)\rangle$, and $\alpha := x+\langle f(x)\rangle$. Then E is a finite field of order p^n. Hence $\alpha^{p^n}=\alpha$. This implies $x^{p^n}-x\in\langle f(x)\rangle$.)

(In class, I took a more advanced route;)

4. Suppose $f(x)\in\mathbb{Z}_p[x]$ is of positive degree. Prove that $f(x)\mid x^{p^k}-x$ for some $k\in\mathbb{Z}^+$ if $f(x)$ is not divisible by the square of an irred. poly. (Hint. Write $f(x)$ as a product of irred.; use problem 3; use $x^{p^m}-x \mid x^{p^n}-x$ if $m\mid n$.)

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5\. Prove that $\mathbb{Z}_3[x]/\langle x^3-x+1\rangle \simeq \mathbb{Z}_3[x]/\langle x^3-x+2\rangle$

(Hint. Prove that both sides are fields of order $3^3=27$.)

6\. Let $\mathbb{Q}[e^{\frac{2\pi i}{n}}]$ be the smallest subfield of $\mathbb{C}$ that contains $\mathbb{Q}$ and $e^{\frac{2\pi i}{n}}$. Prove that $\mathbb{Q}(e^{\frac{2\pi i}{n}})$ is a splitting field of x^n-1 over $\mathbb{Q}$.