Homework 7

Saturday, May 19, 2018 10:45 AM

1. Find all the primes p such that $x+2$ is a factor of $x^6-x^4+x^3-x+1$ in $\mathbb{Z}_p[x]$.

2. Find a zero of x^3-2x+1 in $\mathbb{Z}_5$ and express it as a product of a degree 1 and a degree 2 polynomial.

3. How many degree 2 and degree 3 polynomials with no zeros in $\mathbb{Z}_2[x]$ are there?

4. We are told that x^4-2x^2-2 is irreducible in $\mathbb{Q}[x]$.

(a) Prove that $\mathbb{Q}[x]/\langle x^4-2x^2-2\rangle \simeq \{c_0+c_1\alpha+c_2\alpha^2+c_3\alpha^3 \mid c_i\in\mathbb{Q}\}$ where $\alpha=\sqrt{1+\sqrt{3}}\in\mathbb{R}$.

(b) Write α^{-1} in the form $c_0+c_1\alpha+c_2\alpha^2+c_3\alpha^3$ with $c_i\in\mathbb{Q}$.