Homework 8

Wednesday, May 23, 2018 8:04 AM

1. Prove that the following polynomials are irreducible.

(a) $x^n - 12 \in \mathbb{Q}[x]$ if $n \geq 2$.

(b) $x^3 - 3x^2 + 3x + 4 \in \mathbb{Q}[x]$

(c) We are told that $x^p - x + a$ is irreducible in $\mathbb{Z}_p[x]$ if p is prime and $a \in \mathbb{Z}_p \setminus \{0\}$. Use this fact only for this part of this problem.

$$x^5 - 10x^3 + 25x^2 - 51x + 2017 \in \mathbb{Q}[x].$$

(d) $x^4 + 3x^3 + 27x - 12 \in \mathbb{Q}[x]$.

(e) $x^5 - x + 1 \in \mathbb{Z}_3[x]$

(First show it has no zero in $\mathbb{Z}_3$. Next you can use the following fact without proof: the only monic degree 2 polynomials in $\mathbb{Z}_3[x]$ that do not have a zero in $\mathbb{Z}_3$ are x^2+1, x^2+x-1, and x^2-x-1.)

(f) $x^5 + 2x + 4 \in \mathbb{Q}[x]$

(Use part (e).)

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2. Prove that $\mathbb{Z}_3[x]/\langle x^5-x+1\rangle$ is a field of order 3^5.

(Hint. (1) Use problem 1(e) to show it is a field.

(2) Use long division to show any element of this ring has a unique expression of the form:

$$a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+\langle x^5-x+1\rangle$$

for some $a_0, a_1, a_2, a_3, a_4 \in \mathbb{Z}_3$.)