Homework 1

Wednesday, January 9, 2019 6:27 PM

1. Suppose $R_1, \ldots, R_n$ are rings. Prove that $R_1, \ldots, R_n$ are unital if and only if $R_1 \times \cdots \times R_n$ is unital.

2. Suppose R is a unital ring. An element x of R is called a <u>unit</u> if it has a multiplicative inverse; that means $\exists\, x' \in R$ such that $x x' = x' x = 1_R$.

   Let $R^\times$ be the set of all the units of R.

   (a) Prove that $R^\times$ is closed under multiplication.

   (b) Prove that $(R^\times, \cdot)$ is a group.

   (c) Suppose R_i's are unital rings. Prove that
   $$(R_1 \times \cdots \times R_n)^\times = R_1^\times \times \cdots \times R_n^\times.$$

   (d) Find $(\mathbb{Z} \times \mathbb{Q})^\times$.

3. Show that $\{a + b\sqrt{3} \mid a, b \in \mathbb{Z}\}$ is a subring of $\mathbb{R}$.

4. As in problem 3 one can show $F = \{a + b\sqrt{3} \mid a, b \in \mathbb{Q}\}$ is a ring. Show that $F^\times$ is a field.

Homework 1

Saturday, January 12, 2019 3:14 AM

5. Suppose A is a ring with unity 1. Suppose there is $a_0 \in A$ such that $a_0^2 = 1$. Let $B := \{a_0 r a_0 \mid r \in A\}$. Prove that B is a subring of A.