Name: ____________________

PID: ____________________

Section: ____________________

Question	Points	Score
1	9	
2	9	
3	12	
4	10	
5	10	
Total:	50	

1. Write your Name, PID, and Section on the front page of your exam.
2. No calculators or other electronic devices are allowed during this exam.
3. Read each question carefully, and answer each question completely.
4. Write your solutions clearly in the exam sheet.
5. Show all of your work; no credit will be given for unsupported answers.

1. Find the degree of $(4x^2 - 4x + 1)(6x - 1)$ in $R[x]$ when
   (a) (3 points) $R = \mathbb{Z}_5$.

   (b) (3 points) $R = \mathbb{Z}_4$.

   (c) (3 points) $R = \mathbb{Z}_6$.

2. For each ring determine if $(x-1)(x^2-x+1)$ has more than 3 zeros or not. Justify your answer.

   (a) (3 points) $\mathbb{Z}$.

   (b) (3 points) $\mathbb{Z}_{11}$.

   (c) (3 points) $\mathbb{Z}_{21}$.

3. Determine whether the following polynomials have zeros in $\mathbb{Q}$. Justify your answer.

   (a) (6 points) $x^5 + 12x^4 - 13x^3 + 17x^2 - 15x - 1$.

   (b) (6 points) $x^{7^{2017}} - x^{7^8} + x^{7^{30}} - x + 103$.

4. Prove that the following polynomials are irreducible in $\mathbb{Q}[x]$.

   (a) (5 points) $x^7 + 15x - 45$.

   (b) (5 points) $x^3 - x + 64$.

5. (10 points) Suppose p is a prime. Prove that in $\mathbb{Z}_p[x]$ we have

$$x^p - x = x(x-1)\cdots(x-p+1).$$

Good Luck!