Math 109: The second exam.
Instructor: A. Salehi Golsefidy

Name: . . . .

PID: . . . .

11/15/2016

1. Write your Name and PID on the front of your exam sheet.
2. No calculators or other electronic devices are allowed during this exam.
3. Show all of your work; no credit will be given for unsupported answers.
4. Read each question carefully to avoid spending your time on something that you are not supposed to (re)prove.
5. Ask me or a TA when you are unsure if you are allowed to use certain fact or not.
6. Good luck!

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. (10 points) Suppose A, B, and C are three sets. Prove that,

$$\left.\begin{array}{l}(A \cap B) \subseteq (A \cap C)\\(A \cup B) \subseteq (A \cup C)\end{array}\right\} \Rightarrow B \subseteq C.$$

2. Write the negation of the following propositions:

   (a) (5 points) $\forall L_1, L_2 \in \mathbb{R}, (\forall \varepsilon > 0, |L_1 - L_2| \leq \varepsilon) \Rightarrow L_1 = L_2)$.

   (b) (5 points) $\forall \varepsilon > 0, \exists \delta > 0, \forall x \in \mathbb{R}, (|x - 1| < \delta \Rightarrow |x^3 - 1| < \varepsilon)$.

3. Suppose $f : X \to Y$ and $g : Y \to Z$ are two functions and $g \circ f$ is a bijection.

   (a) (5 points) Prove that f is injective.

   (b) (5 points) Prove that g is surjective.

4. (10 points) Let X be a non-empty set. For any subset A of X, let $\mathbb{1}_A$ be the characteristic function of A; that means

$$\mathbb{1}_A : X \to \{0, 1\}, \qquad \mathbb{1}_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

Suppose A and B are two subsets of X. Prove that,

$$(\forall x \in X, \mathbb{1}_A(x) \leq \mathbb{1}_B(x)) \iff A \subseteq B.$$