

We take a naive approach towards set theory, and on purpose leave some terms undefined.

What is a set? . A set is a well-defined collection of objects, a box containing certain objects.

Let's start with special sets:

The set of integers: \mathbb{Z} (Zahlen)

The set of rational numbers: \mathbb{Q}

The set of real numbers: \mathbb{R}

The set of complex numbers: \mathbb{C}

Objects in a set are called its **elements** or **members**; we write $a \in A$ to say a is in A or a is an element of A .

Ex. $1/2 \in \mathbb{Q}$; $1/2 \notin \mathbb{Z}$; $i \in \mathbb{C}$; $i \notin \mathbb{R}$.

We need more examples in order to understand sets better.

A set can be given in various ways.

List the elements.

Ex. $A = \{1, 2\}$

$1 \in A, 3 \notin A$

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Ex. $B = \{1, \{1, 2\}\}$

$1 \in B, 2 \notin B$

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$$2+3 \neq \pi$$

$$2+1+3 = 0, 2+3 \neq 0$$

Ex. $C = \{ \}$ (The empty box;
The set with no elements.)

- The empty set.
- It is also denoted by \emptyset .

$$\{ \} \notin C$$

Ex. Can you tell me a set that contains 1?

How about a? How about $\{ \}$?

$$\{1\}; \{a\}; \{\{\}\}$$

Ex. How many elements does $\{\{\}\}$ have?

How about $\{\}$?

Two sets are equal if they have collection of members.

(Repeating an element does NOT change the set.)

Ex. How many elements do the following sets have?

$$\{1, 2, 2, 3\}$$

3 its elements are 1, 2, 3

$$\{\{1, 2\}\}$$

1 it has only one element $\{1, 2\}$

$$\{1, 2, \{1, 2\}\}$$

3 it has three elements 1, 2, $\{1, 2\}$

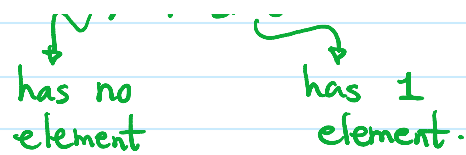
$$\{\emptyset, \{\}\}$$

1 Notice that $\emptyset = \{ \}$

$$\{\emptyset, \{\emptyset\}\}$$

2 it has two elements $\emptyset, \{\emptyset\}$.

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Another way to construct a set is by giving the conditions of membership.

Ex. $\{n \in \mathbb{Z} \mid n \text{ is odd}\}$ (the set of odd numbers)

. $\{n \in \mathbb{Z} \mid n \geq 3, n \text{ is odd}\}$

. $\{z \in \mathbb{C} \mid \text{Im}(z) > 0\}$ (upper-half plane)

Constructing the elements of the set

Ex. The set of even numbers = $\{2k \mid k \in \mathbb{Z}\}$.

. The set of odd numbers = $\{2k+1 \mid k \in \mathbb{Z}\}$.

(Recall. Two sets are equal if they are collections of the same objects.)

So the second claim is equivalent to

$$m \text{ is odd} \iff m = 2k+1 \text{ for some integer } k.$$

. $\{x \in \mathbb{R} \mid x^2+1=0\} = \emptyset$

. $\{x \in \mathbb{R} \mid x^2-1=0\} = \{-1, 1\}$.

Def. We say A is a subset of B if

any element of A is an element of B .

$A \subseteq B$

Ex. • $\{1\} \subseteq \{1, 2\}$

• $\{\{1\}\} \not\subseteq \{1, 2\}$

To show $A \not\subseteq B$, it is enough to find x st.

$$x \in A \quad \wedge \quad x \notin B.$$

$$\neg (x \in A \Rightarrow x \in B) \equiv x \in A \wedge x \notin B.$$

$\{1\} \in \{\{1\}\}$, but $\{1\} \notin \{1, 2\}$.

(we have seen that elements of $\{1, 2\}$ are 1 and 2, and 1 is NOT the same as $\{1\}$.)

• $\{\} \subseteq \{1, 2\}$

If not, then we should be able to find

$$x \in \{\} \quad \wedge \quad x \notin \{1, 2\};$$

but the empty set $\{\}$ has no elements.

• For any set A , $\emptyset \subseteq A$ and $A \subseteq A$.