We take a naïve approach towards set theory, and on purpose leave some terms undefined.

What is a set? . A set is a well-defined collection of objects; a box containing certain objects.

Let's start with special sets:

The set of integers: $\mathbb{Z}$ (Zahlen)

The set of rational numbers: $\mathbb{Q}$

The set of real numbers: $\mathbb{R}$

The set of complex numbers: $\mathbb{C}$

Objects in a set are called its elements or members; we write $a \in A$ to say a is in A or a is an element of A.

Ex. $1/2 \in \mathbb{Q}$; $1/2 \notin \mathbb{Z}$; $i \in \mathbb{C}$; $i \notin \mathbb{R}$.

We need more examples in order to understand sets better.

A set can be given in various ways.

List the elements.

Ex. $A = \{1, 2\}$

$1 \in A$, $3 \notin A$

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Ex. $B = \{1, \{1,2\}\}$

$1 \in B$, $2 \notin B$

$\{1\} \notin A$ | $\{1,2\} \subseteq B$, $\{1\} \not\subseteq B$

Ex. $C = \{\ \}$ (The empty box; The set with no elements.)

- The empty set.
- It is also denoted by $\emptyset$.

$\{\ \} \notin C$

Ex. Can you tell me a set that contains 1?

How about a? How about $\{\ \}$?

$\{1\}$; $\{a\}$; $\{\{\ \}\}$.

Ex. How many elements does $\{\{\ \}\}$ have?

How about $\{\ \}$?

Two sets are equal if they have collection of members.

(Repeating an element does NOT change the set.)

Ex. How many elements do the following sets have?

$\{1,2,2,3\}$ 3 its elements are 1, 2, 3

$\{\{1,2\}\}$ 1 it has only one element $\{1,2\}$

$\{1,2,\{1,2\}\}$ 3 it has three elements 1, 2, $\{1,2\}$

$\{\emptyset, \{\ \}\}$ 1 Notice that $\emptyset = \{\ \}$

$\{\emptyset, \{\emptyset\}\}$ 2 it has two elements $\emptyset$, $\{\emptyset\}$.

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has no element

has 1 element.

Another way to construct a set is by giving the conditions of membership.

<u>Ex.</u> . $\{n \in \mathbb{Z} \mid n \text{ is odd}\}$ (the set of odd numbers)

. $\{n \in \mathbb{Z} \mid n \geq 3 , n \text{ is odd}\}$

. $\{z \in \mathbb{C} \mid \operatorname{Im}(z) > 0\}$ (upper-half plane)

Constructing the elements of the set

<u>Ex.</u> . The set of even numbers $= \{2k \mid k \in \mathbb{Z}\}$.

. The set of odd numbers $= \{2k+1 \mid k \in \mathbb{Z}\}$.

(<u>Recall</u>. Two sets are equal if they are collections of the same objects.)

So the second claim is equivalent to

$$m \text{ is odd} \iff m = 2k+1 \text{ for some integer } k.$$

. $\{x \in \mathbb{R} \mid x^2+1=0\} = \emptyset$

. $\{x \in \mathbb{R} \mid x^2-1=0\} = \{-1, 1\}$.

<u>Def.</u> We say A is a subset of B if any element of A is an element of B.

A B

<u>Ex.</u> • $\{1\} \subseteq \{1,2\}$

• $\{\{1\}\} \not\subseteq \{1,2\}$

To show $A \not\subseteq B$, it is enough to find x s.t.

$$x \in A \ \wedge \ x \notin B.$$

$$\neg (x \in A \Rightarrow x \in B) \equiv x \in A \wedge x \notin B.$$

$\{1\} \in \{\{1\}\}$, but $\{1\} \notin \{1,2\}$.

(we have seen that elements of $\{1,2\}$ are 1 and 2, and 1 is NOT the same as $\{1\}$.)

• $\{\ \} \subseteq \{1,2\}$

If not, then we should be able to find

$x \in \{\ \} \wedge x \notin \{1,2\}$;

but the empty set $\{\ \}$ has <u>no</u> elements.

. For any set A, $\emptyset \subseteq A$ and $A \subseteq A$.