1. Write down the negation of the following statements:

(a) $\forall \varepsilon > 0, \ \exists \delta > 0, \ |x-1| < \delta \Rightarrow |x^2-1| < \varepsilon.$

(b) $\forall \varepsilon > 0, \forall x \in \mathbb{R}, \ \exists n \in \mathbb{Z}, \ |x-n| < \varepsilon$

(c) Let α be an irrational number, i.e. $\alpha \in \mathbb{R} \setminus \mathbb{Q}$.

$$\forall \varepsilon > 0, \ \forall x \in \mathbb{R}, \ \exists m, n \in \mathbb{Z}, \ |x - m - n\alpha| < \varepsilon.$$

(d) $\forall a \in \mathbb{Z}, \ \forall \ b \in \mathbb{Z}^{>0}$, there is a unique pair (q, r) of integers such that

$$a = bq + r \quad \text{and} \quad 0 \leq r < b.$$

(you are not allowed to use $\nexists$.)

2. (a) Prove or disprove: $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, \ y^2 > 2015 + x$

(b) Prove or disprove: $\exists x \in \mathbb{R}, \ \forall y \in \mathbb{R}, \ y^3 > 2015 + x$

(c) Prove or disprove: $\forall \varepsilon > 0, \exists N \in \mathbb{Z}^{>0}, n \geq N \Rightarrow \frac{1000}{n} < \varepsilon.$

(For part (c), you are allowed to use the following:

$$\forall x \in \mathbb{R}, \exists n \in \mathbb{Z}, \ x < n.)$$

3. Prove that $\lim\limits_{x \to 2} x^3 = 8.$

4. Prove that

$$\forall n \in \mathbb{Z}^{>1}, \left(\left(\nexists m \in \mathbb{Z},\ 1 < m \leq \sqrt{n} \wedge m \mid n \right) \Rightarrow n \text{ is prime} \right.$$

(Hint. Prove by contradiction.)