Problem set 1

Tuesday, September 29, 2015 1:38 PM

1. Construct truth tables for the following propositional forms.

(i) $P \wedge (Q \vee R)$ (iii) $\neg(P \Rightarrow Q)$

(ii) $(P \wedge Q) \vee (P \wedge R)$ (iv) $(\neg P) \wedge Q$

2. Use the first problem to deduce that

(i) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$ (distributive law)

(ii) $\neg(P \rightarrow Q) \equiv P \wedge (\neg Q)$

3. Prove that for any positive real numbers a and b we have $\sqrt{ab} \geq \min\{a, b\}$.

4. Prove that for any real numbers a and b we have

$$|a+b| \leq |a| + |b|.$$

(Hint. $x^2 \leq y^2 \iff |x| \leq |y|$ and $z \leq |z|$.)

5. Prove that for all integers n,

$$4(n^2+n+1) - 3n^2$$

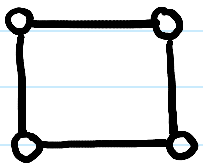
is a perfect square.

6. We would like to color each circle in a way that two connected circles have different colors.

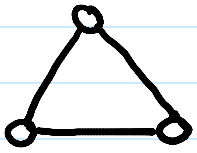
what is the minimum number of needed colors?

Justify your answer.

(i)



(ii)



(iii)