Problem set 1

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1. Prove that $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

   and $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

   (These are called distribution laws.)

2. Prove that $(P \vee Q) \Rightarrow R \equiv (P \Rightarrow R) \wedge (Q \Rightarrow R)$

   (This is essentially the case-by-case proof: if there are two possibilities for the hypothesis, one has to deduce the conclusion from each case separately.)

   Can you prove the above equivalency without using truth table?

3. Show that $P \Rightarrow Q \not\equiv Q \Rightarrow P$.

4. ⓐ Prove that for any real number a we have
   $$|a| \geq a.$$
   ⓑ Prove that for any real number b we have
   $$|b|^2 = b^2.$$
   ⓒ Prove that for any real numbers c and d we have
   $$|c| + |d| \geq |c+d|.$$

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5. Prove that $P \Rightarrow (Q \vee R) \equiv (P \wedge (\neg Q)) \Rightarrow R$

$$\equiv (P \wedge (\neg Q) \wedge (\neg R)) \Rightarrow \perp$$

Can you prove this without using truth table?

6. Suppose d, m_1, m_2, n_1, and n_2 are integers. Prove that

$$(d \mid m_1 - m_2 \wedge d \mid n_1 - n_2) \Rightarrow (d \mid (m_1+n_1)-(m_2+n_2) \wedge d \mid m_1n_1 - m_2n_2)$$

Instead of "and" sometimes we use $\Big\{$ or $\Big\}$. So the above problem can be written as

$$\left.\begin{array}{l} d \mid m_1 - m_2 \\ d \mid n_1 - n_2 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} d \mid (m_1+n_1)-(m_2+n_2) \\ d \mid m_1n_1 - m_2n_2 . \end{array}\right.$$