Lecture 1: Proposition

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In this course you are supposed to learn

- How to listen to a proof and understand it.
- How to read a proof and understand it.
- How to produce a proof and communicate your thoughts.

We will use different parts of mathematics to achieve this goal. We start with Propositional Logic, introduce quantifiers, use basic ideas from game theory, discuss $\varepsilon-\delta$ definition of limit, study a little bit of arithmetic. The key to success, however, is doing lots of exercises.

Mathematical Language

Proposition is a sentence that is either true or false (not at the same time!).

Ex. $1+1$ is NOT a proposition. A proposition has to claim something. That claim might be true or false. A sentence with no claim is not a proposition.

Ex. $1+1=3$ is a proposition. It is a false proposition.

Ex. $m=1$. It is NOT a proposition. We do not know where m lives and in what capacity should we look for it.

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At the same, I should add that one often sees a sentence similar to "Let $m=1$" in math articles or books. This sentence is NOT a proposition as it makes no claim.

Ex. For any rational number m, $m=1$.

This is a proposition. In fact, this is a false proposition.

To see it is false, it is enough to present a counter-example. Since this proprosition claims that certain property should hold for any rational number, it is enough to find a single rational number which does NOT satisfy the claimed property to get that it is a false proposition. (I agree! It became a very long and more complicated than it should have been, but hopefully you have got point.)

2 is a rational number and $2 \neq 1$.

So 2 is a counter-example.

Ex. If m is integer and $0<m<2$, then $m=1$.

This is a true proposition.

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Ex. $x^2 \geq 0$

It is NOT a proposition. It does NOT make any claim: where x lives and it what capacity should we look for it

For instance we can make it into a proposition, as follows:

For any real number x, $x^2 \geq 0$.

There is a complex number x such that $x^2 \geq 0$ does not hold.

- $x^2 \geq 0$ and $m=1$ are called predicates and x and m in these sentences are called free variables.

- Adding quantifiers and determining "universes" for free variables of a predicate makes it into a proposition.

Ex. If m is a rational number and $0 < m < 2$, then $m=1$.

It is a false proposition. To show a conditional sentence is false one has to find an example where the hypothesis holds and at the same time the conclusion fails.

$\frac{1}{2}$ is a rational number and $0 < \frac{1}{2} < 2$, and $\frac{1}{2} \neq 1$.

So $\frac{1}{2}$ is a counter-example.

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Let's try to write down our general method of finding the truth value of a conditional statement.

If <u>something</u>, then <u>something</u>.

This is too ambiguous. So we use variables instead of "somethings".

If H, then C.

Here H and C stand for two propositions. (It is similar to calculus where variables are used instead of numbers, here variables are used instead of propositions.) What we said about the truth-value of this conditional statement can be summeraized in the following table

H	C	if H, then C
T	T	T
T	F	F
F	T	T
F	F	T

all the possible truth-value combinations

Should be false only when H holds and C fails

As you saw in the previous examples, we can <u>connect</u> propositions to form new propositions.

Suppose P and Q are two propositions.

Conjunction. P and Q.

It is denoted by $P \wedge Q$.

Disjunction. P or Q.

It is denoted by $P \vee Q$.

Conditional sentence or implication

If P, then Q.

P implies Q.

P is sufficient for Q.

Q is necessary for P.

It is denoted by

$P \Rightarrow Q$.

Next time we will discuss the truth-value table of these connectives.