Name: ____________________

PID: ____________________

Section: ____________________

Question	Points	Score
1	10	
2	10	
3	10	
Total:	30	

1. Write your Name and PID, on the front page of your exam.
2. Read each question carefully, and answer each question completely.
3. Write your solutions clearly in the exam sheet.
4. Show all of your work; no credit will be given for unsupported answers.
5. You may use the result of one part of the problem in the proof of a later part, even you do not complete the earlier part.
6. You may use major theorems *proved* in class, but not if the whole point of the problem is reproduce the proof of a theorem proved in class or the textbook. Similarly, quote the result of a homework exercise only if the result of the exercise is a fundamental fact and reproducing the result of the exercise is not the main point of the problem.

1. (10 points) Suppose G is a finite group, and P is a Sylow p-subgroup. Suppose $|\mathrm{Syl}_p(G)| \geq [G : P]$, where $\mathrm{Syl}_p(G)$ is the set of all the Sylow p-subgroups of G. Prove that for any $g \in G$ we have $g \in \langle P, gPg^{-1} \rangle$.

2. Suppose G is a non-abelian finite group, $Z(G)$ is its center, and $G/Z(G)$ is a p-group.

   (a) (5 points) Prove that G has a unique Sylow p-subgroup P. (Hint: Think about $[G : N_G(P)Z(G)]$.)

(b) (5 points) Prove that $p||Z(G)|$.

3. Suppose G is a group of order 56. Let P_2 be a Sylow 2-subgroup of G, and P_7 be a Sylow 7-subgroup of G.

   (a) (5 points) Prove that either P_2 is normal in G or P_7 is normal in G.

(b) (5 points) Show that there are at least two non-isomorphic non-abelian groups of order 56. (Hint: You are allowed to use without proof that $|\operatorname{GL}_3(\mathbb{Z}/2\mathbb{Z})| = (7)(24)$.)

Good Luck!