Lecture 1: Vectors

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In the first lecture, we recalled what a <u>vector</u> is.

How a vector can be <u>described</u> or <u>represented</u>.

How we can do <u>algebra</u> with vectors, and

how the algebra can be visualized.

<u>Conceptual</u>: A vector carries two information

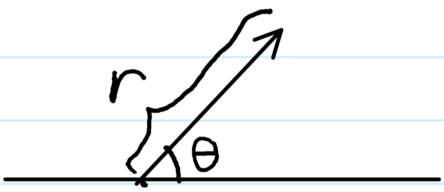
(1) direction

(2) magnitude or length.

For example you can think about <u>movement</u>.

- Go $\underbrace{\text{towards north}}_{(1)}$ for $\underbrace{\text{5 miles.}}_{(2)}$
- Go $\underbrace{\text{towards north-west}}_{(1)}$ for $\underbrace{\text{1 mile.}}_{(2)}$

We can use angle to talk about a general direction in a plane.



Important remark. The <u>movement itself</u> is what we are interested in, and <u>NOT</u> the initial and terminal points.

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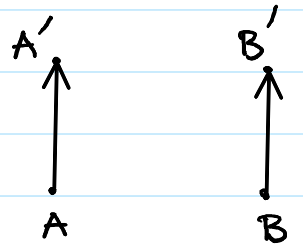
For two people from the points A and B can move towards north for 1 mile, and reach to the points A' and B'.

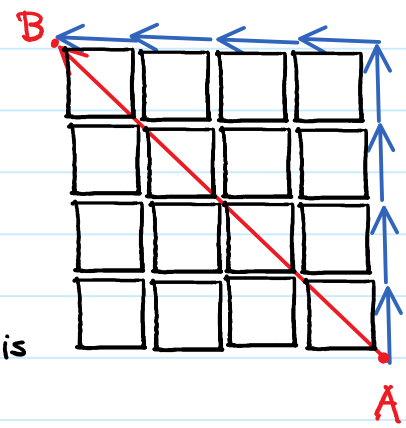


They are doing the same movement. So

$$\overrightarrow{AA'} = \overrightarrow{BB'}.$$

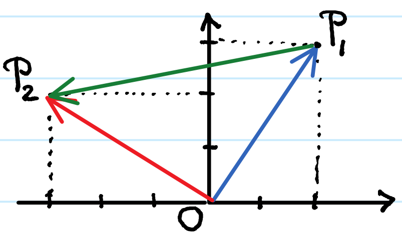
(same direction and same length, but different initial points.)

. If you are in Manhattan and ask for direction, you do not expect to hear "go towards north west for 1 miles"! Rather you'd like to hear "go north for 4 blocks and go west for 4 blocks".



In mathematical language, we can use xy-axis to describe a vector.

Ex.



$\overrightarrow{OP_1} = (2,3)$

$\overrightarrow{OP_2} = (-3,2)$

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Ex. What is $\overrightarrow{P_1P_2}$ in the previous example?

Looking at the picture and thinking about the movement from P_1 to P_2 we see that one should go 5 steps towards the negative direction of the x-axis and 1 step towards the negative direction of the y-axis. So

$$\overrightarrow{P_1P_2} = (-5, -1).$$

General Rule. $A_1 = (x_1, y_1)$ and $A_2 = (x_2, y_2)$

Then $\overrightarrow{A_1A_2} = (x_2 - x_1, y_2 - y_1)$

Coordinates of the terminal point − Coordinates of the initial point

Ex. Compute the components of $\overrightarrow{AB}$ where $A = (1, -1)$ and $B = (4, 2)$.

Solution.
$$\begin{aligned}\overrightarrow{AB} &= (x_B - x_A, y_B - y_A)\\ &= (4 - 1, 2 - (-1))\\ &= (3, 3).\end{aligned}$$

How can we go back and forth between these two ways of describing a vector?

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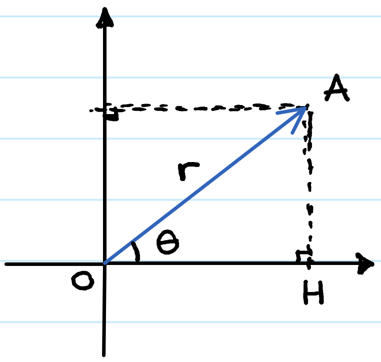
Using components is perfect for doing computations and the geometric interpretation is perfect for visualizing and getting a good intution about the algebraic manipulation of vectors.

Direction and length $\rightsquigarrow$ Coordinates



Suppose length of $\vec{OA}$ is r and it makes angle θ with the positive direction of the x-axis.

Then, since the triangle OAH is a right triangle,

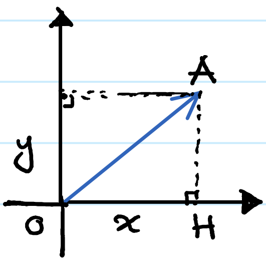
$$|\cos\theta| = \frac{|OH|}{|OA|}$$

length of the segment OH.

So $|OH| = r\,|\cos\theta|$. In fact one can see that the x-compon. of $\vec{OA}$ is $r\cos\theta$. Similarly one can see that the y-comp. of $\vec{OA}$ is $r\sin\theta$. So

$$\boxed{\vec{OA} = (r\cos\theta,\ r\sin\theta)}$$

Coordinates $\rightsquigarrow$ Direction and length.



Since $\overset{\triangle}{OAH}$ is a right triangle,

length of segments

by Pythagorean Theorem $|OA|^2 = |OH|^2 + |AH|^2$

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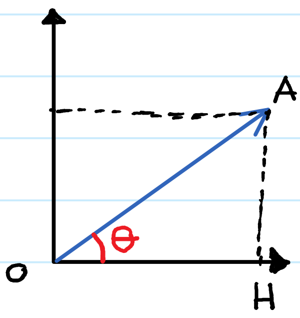
Length of a vector $\vec{v}$ is denoted by $\|\vec{v}\|$. It is also called <u>norm of $\vec{v}$</u> or <u>magnitude of $\vec{v}$</u>. So

$$\|\vec{OA}\|^2 = |x|^2+|y|^2 = x^2+y^2.$$ Hence

$$\boxed{\text{If } \vec{v}=(x,y), \text{ then } \|\vec{v}\|=\sqrt{x^2+y^2}.}$$

Using the above mentioned equations for $\cos\theta$ and $\sin\theta$ we get:

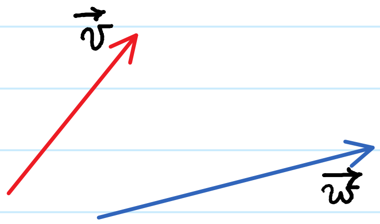


$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

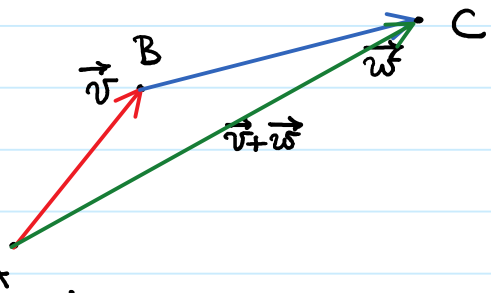
$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

$$\tan\theta = \frac{y}{x}$$

. Having two vectors $\vec{v}$ and $\vec{w}$ and thinking about them as movements, we can "do them" one after another.



Starting from a point A, first we move according to $\vec{v}$ and then we continue according to $\vec{w}$.



Instead we can directly go to C. We denote this by +. $\vec{AB}+\vec{BC}=\vec{AC}$

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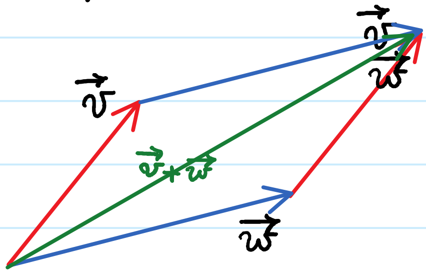
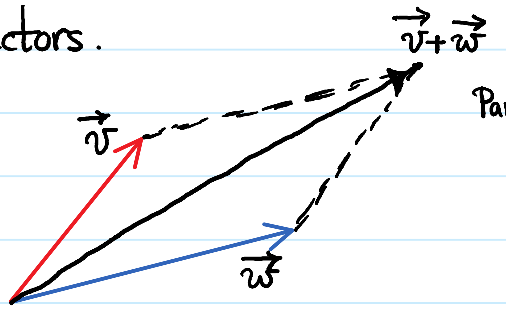
In fact, one can see that $(x_1,y_1)+(x_2,y_2)=(x_1+x_2,y_1+y_2)$.

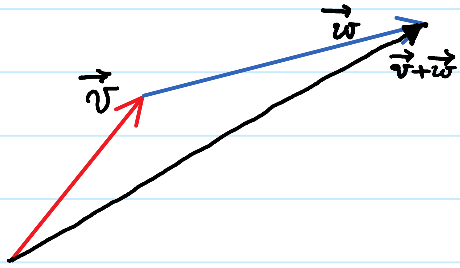
In particular it does not matter in which order we apply $\vec{v}$ and $\vec{w}$; we end up to the same end point. Geometrically we have:



From this picture, we can get two rules to visualize addition of two vectors.



Parallelogram law



Triangle law