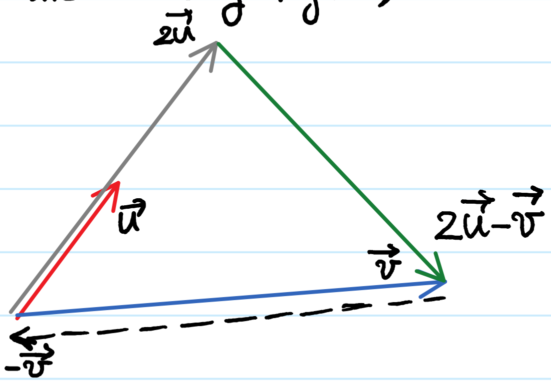
Lecture 2: Geometric understanding

Monday, September 26, 2016 8:28 AM

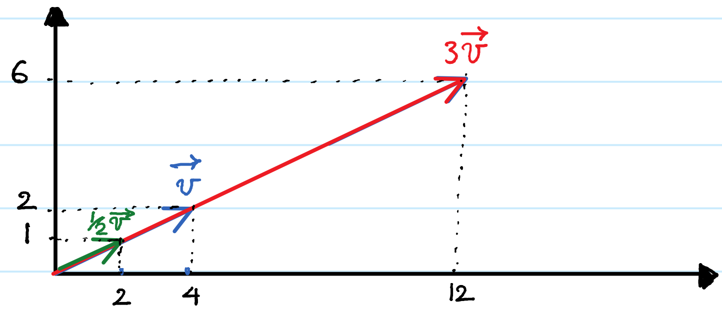
In the previous lecture we saw the parallelogram law, let's start todays lecture with a visualization example:

<u>Ex.</u> In the following figure, sketch $2\vec{u}-\vec{v}$.



. Scalar multiplication: Let c be a number and $\vec{v}=(x,y)$ be a vector. Then $c\vec{v}=(cx,cy)$.

<u>Ex.</u> Sketch $\vec{v}=(4,2)$, $3\vec{v}$, and $\frac{1}{2}\vec{v}$.



<u>Definition</u> Two vectors $\vec{v}$ and $\vec{w}$ are called parallel if $\vec{v}=c\vec{w}$ for some constant c.

[Geometrically this means the underlying lines of these vectors are parallel.]

Lecture 2: 3D vectors and standard basis

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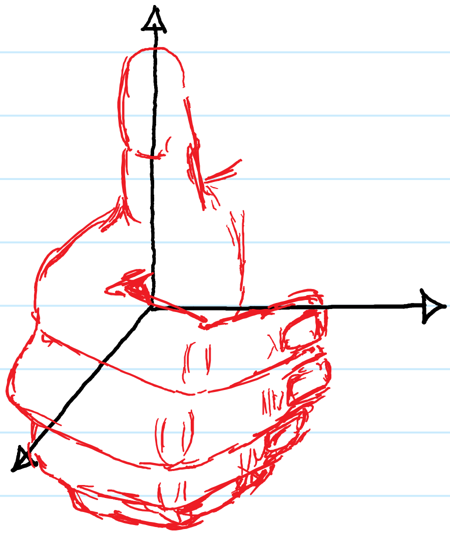
When working with a moving particle in space, for its position, or velocity, we need three numbers. In many applications we are dealing with an entity that cannot be described only with 2 or 3 numbers, in which case one needs to work with "higher dimensional" spaces (as you see for instance in a linear algebra course.) In this coure we work with either 2D or 3D vectors.

We can visualize it using x-axis, y-axis, and z-axis.

These are three pairwise orthogonal directed lines. The positive directions of these lines follow the right hand rule.



Let $\vec{i}=(1,0,0)$, $\vec{j}=(0,1,0)$, and $\vec{k}=(0,0,1)$. Then

$$x\vec{i}+y\vec{j}+z\vec{k}=x(1,0,0)+y(0,1,0)+z(0,0,1)$$

Lecture 2: Standard basis

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So $x\vec{i} + y\vec{j} + z\vec{k} = (x, y, z)$.

So any vector can be uniquely written as a linear combination of $\vec{i}$, $\vec{j}$, and $\vec{k}$.

<u>Ex</u>. Compute components of $(2\vec{i} + \vec{k}) - 2(0, 1, 2)$.

<u>Solution</u>. $2\vec{i} + \vec{k} = 2(1,0,0) + (0,0,1)$

$$= (2, 0, 1).$$

alternatively $2\vec{i} + \vec{k} = 2\vec{i} + 0\vec{j} + 1\vec{k}$

$$= (2, 0, 1)$$

So $(2\vec{i} + \vec{k}) - 2(0, 1, 2) = (2, 0, 1) - (0, 2, 4)$

$$= (2, -2, -3).$$

Lecture 2: Moving particle

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Moving particle with constant velocity $\vec{v}_0$.

. Let's think about $\vec{v}_0$ as the velocity of a particle. Suppose that the particle is at the point $P_0=(x_0, y_0, z_0)$. Then after $\underline{t}$ seconds where is the particle?

By the definition, velocity is the rate of change of the positional vector. What does this mean? Suppose this particle is at the point P_t at time t. Then

$$\text{change of positional vector} = \overrightarrow{OP_t} - \overrightarrow{OP_0}$$

And <u>average</u> change is $\dfrac{\overrightarrow{OP_t}-\overrightarrow{OP_0}}{t}$. Since, by our assumption the velocity is constant, we have $\vec{v}_0 = \frac{1}{t}(\overrightarrow{OP_t}-\overrightarrow{OP_0})$ which implies

$$\boxed{\overrightarrow{OP_t} = \overrightarrow{OP_0} + t\,\vec{v}_0}$$

Positional vector of a particle with initial point P_0 and constant velocity $\vec{v}_0$.