Lecture 3: Parametrization of a line

Monday, September 26, 2016 8:48 AM

Let me recall that in the previous lecture we saw

A moving particle with the initial position $P_0 = (x_0, y_0, z_0)$ and constant velocity $\vec{v}_0 = (a, b, c)$ reaches to the point P_t at time t where

$$\overrightarrow{OP_t} = t\,\vec{v}_0 + \overrightarrow{OP_0}.$$

Application to Geometry.

As we keep track of a particle with constant velocity, we realize that it is moving on a line which passes through P_0 and is parallel to $\vec{v}_0$. So we get the following:

Line passing through $P_0 = (x_0, y_0, z_0)$ and parallel to the vector $\vec{v}_0 = (a, b, c)$ is given by

$$\overrightarrow{OP} = t\,\vec{v}_0 + \overrightarrow{OP_0}$$ vector parametric equation

$$\begin{cases} x = at + x_0 \\ y = bt + y_0 \\ z = ct + z_0 \end{cases}$$ parametric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$ with the understanding that, if $a = 0$, then $x = x_0$ or similarly $b = 0$ implies $y = y_0$, $c = 0$ implies $z = z_0$

Lecture 3: Line

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Ex. Find parametric equation of a line which passes throug the point $P_0=(1,2,0)$ and it is parallel to the vector $\overrightarrow{AB}$ where $A=(1,1,0)$ and $B=(1,0,1)$.

Solution. A vector-parametric equation of this line is

$$\overrightarrow{OP}=\overrightarrow{OP_0}+t\,\overrightarrow{AB}.$$

So we need to compute $\overrightarrow{AB}$.

$$\overrightarrow{AB}=\underbrace{(1,0,1)}_{\text{terminal}}-\underbrace{(1,1,0)}_{\text{initial}}=(0,-1,1)$$

So $\begin{cases} x=1 \\ y=2-t \\ z=0+t=t \end{cases}$ parametric equation of this line.

Ex. Find parametric equation of a line which passes through the points $A=(1,1,0)$ and $B=(1,0,1)$.

Solution. So this line passes through A and it is parallel to $\overrightarrow{AB}$. So its vector-parametric equation is

$$\overrightarrow{OP}=\overrightarrow{OA}+t\,\overrightarrow{AB}=(1,1,0)+t\underbrace{(0,-1,1)}_{\text{as we have seen above}}$$

$$\Rightarrow\begin{cases} x=1 \\ y=1-t \\ z=t \end{cases}$$

Lecture 3: Parametrization of a segment

Wednesday, September 28, 2016 8:29 AM

. How can we parametrize a segment AB?

. It is clearly part of the line which passes through the points A and B so any point P on this segment is of the form $\overrightarrow{OP} = \overrightarrow{OA} + t\,\overrightarrow{AB}$.

. What range of t gives us points on the segment AB?

. Think about a moving particle with initial point A and constant velocity $\overrightarrow{AB}$. For what interval of time is this particle moving between A and B?

At $t=0$ it is at A, and at $t=1$ its positional vector is $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$. So for $0 \le t \le 1$ it is on the segment AB. Hence

$$\overrightarrow{OP} = \overrightarrow{OA} + t\,\overrightarrow{AB}$$

$$= \overrightarrow{OA} + t\,(\overrightarrow{OB} - \overrightarrow{OA}) = (1-t)\,\overrightarrow{OA} + t\,\overrightarrow{OB}$$

for $0 \le t \le 1$ is a parametrization of the segment AB

Lecture 3: Midpoint, inner product

Wednesday, September 28, 2016 8:40 AM

Let's also notice at $\underline{t=1/2}$, the above particle reaches to the midpoint M of the segment AB. So

$$\boxed{\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OB}}.$$

. One of the key algebraic operations between vectors is the dot-product of two vectors:

$$\vec{v}_1 = (x_1, y_1, z_1) \quad \text{and} \quad \vec{v}_2 = (x_2, y_2, z_2)$$

Then $\vec{v}_1 \cdot \vec{v}_2 = x_1 x_2 + y_1 y_2 + z_1 z_2$.

Warning 1. Dot-product $\vec{v}_1 \cdot \vec{v}_2$ is a number. It is NOT a 3D-vector.

Warning 2. Please do NOT write

$$(x_1, y_1, z_1) \cdot (x_2, y_2, z_2) = x_1 x_2 \quad y_1 y_2 \quad z_1 z_2$$

It has NO meaning.

Ex. Let $\vec{v} = (1, 2, 3)$ and $\vec{w} = (-1, 0, 1)$. Then

$\vec{v} \cdot \vec{w} = (1)(-1) + (2)(0) + (3)(1) = 2$.

$\vec{v} \cdot \vec{i} = 1$ first component of $\vec{v}$

$\vec{w} \cdot \vec{j} = 0$.