

Name: \_\_\_\_\_ Student Number: \_\_\_\_\_

**Math 20C.**  
**Midterm Exam 2**  
**July 23, 2004**

*Read each question carefully, and answer each question completely.*  
*Show all of your work. No credit will be given for unsupported answers.*  
*Write your solutions clearly and legibly. No credit will be given for illegible solutions.*

1. (8 points)

Consider the function  $f(x, y, z) = \sqrt{x + 2yz}$ .

(a) Find the gradient of  $f(x, y, z)$ .

(b) Find the directional derivative of  $f$  at  $(0, 2, 1)$  in the direction given by  $\langle 0, 3, 4 \rangle$ .

(c) Find the maximum rate of change of  $f$  at the point  $(0, 2, 1)$ .

#	Score
1	
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2. (8 points)

Find any value of the constant  $a$  such that the function  $f(x, y) = e^{-ax} \cos(y) - e^{-y} \cos(x)$  is solution of Laplace's equation  $f_{xx} + f_{yy} = 0$ .

3. (8 points)

Let  $f(x, y) = 12xy - 2x^3 - 3y^2$ .

- (a) Find all the critical (stationary) points of  $f$ .
- (b) For each critical point of  $f$ , determine whether  $f$  has a local maximum, local minimum, or saddle point at that point.

4. (8 points)

Use Lagrange multipliers to find the maximum and minimum values of the function  $f(x, y) = x^2 + y^2$  subject to the constraint  $\frac{1}{4}x^2 + \frac{1}{9}y^2 = 1$ .