Math 21C Final, Fall 02, Lindblad.

1. A particle moves with position vector given by $\mathbf{r}(t) = t^2\,\mathbf{i} + (1-t^2)\,\mathbf{j} + t^3\mathbf{k}$.
(a) Find the equation of the tangent line to the curve at $t = 1$!
(b) What distance does the particle travel between time $t = 0$ and $t = 1$?

2. Given the three points $P(0,1,1)$, $Q(0,2,0)$ and $R(2,1,0)$.
(a) Find the equation of the plane containing the three points!
(b) Find the Area of the triangle with the three points as corners!
(c) What are the cosines of the angles at the three corners of the triangle in (b)?

3. The temperature of a metal ball centered at the origin is given by $T = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$.
(a) Find the rate of change of T at $(1,2,2)$ in the direction toward the point $(2,1,3)$
(b) Explain why for any point (x,y,z) in the ball, the direction of greatest increase in the temperature is toward the center of the ball.

4. (a) Find the points on the ellipsoid $4x^2 + y^2 + z^2 = 4$ where the tangent plane is parallel to the plane $x + 2y - z = 0$.
(b) Find the equation for the tangent plane at these points.

5. (a) Find the critical points of $f(x,y) = x^2 + 5y^2 + 3xy + 8$ and determine if they are local max, min or saddle points.
(b) Find the max and min of $f(x,y)$ on the set $g(x,y) = x^2 + y^2 = 4$.
(c) Find the absolute max and min of $f(x,y)$ over the set $D = \{(x,y)|g(x,y) \leq 4\}$.

6. UPS charges according to weight as well as size, for sending a box. If $x, y, z \geq 0$ are the lengths of the edges of a rectangular box, then in order that the package should not be over sized we must have $G(x,y,z) = 2x + 2y + z \leq 120$ inches. Find the maximum of the volume $V(x,y,z) = xyz$ for a package that is not over sized.

7. (a) Write the iterated integral $\displaystyle\int_0^1 \int_1^{1/y} x^2 e^{-x^2}\,dx\,dy$ as a double integral over some unbounded domain D in the x-y plane.
(b) Evaluate the integral in (a) by changing the order of integration.

8. Find $\displaystyle\iint_R (1+y)\cos(x^2+y^2)\,dA$, where $R = \{(x,y);\ 1 \leq x^2 + y^2 \leq 4\}$.

9. Find the area of the part of the plane $x + 2y + z = 8$ above the region $D = \{(x,y);\ 0 \leq x \leq 1,\ 2x^2 \leq y \leq 1 + x^2\}$, i.e. of the surface $S = \{(x,y,z);\ (x,y) \in D,\ x + 2y + z = 8\}$.

10. Let D be the triangle in the x-y plane with vertices $(0,0)$, $(0,4)$ and $(1,2)$. Write the volume of the solid E, below the plane $z = x - y + 20$ and above the triangle D, as a double integral over D and evaluate it.
(i.e. $E = \{(x,y,z)|\ 0 \leq z \leq x - y + 20,\ (x,y) \in D\}$.)