Print Name: ____________________ Student Number: ____________

Section Time: ____________

Math 20C.
Midterm Exam 2
November 21, 2005

Read each question carefully, and answer each question completely.
Show all of your work. No credit will be given for unsupported answers.
Write your solutions clearly and legibly. No credit will be given for illegible solutions.

1. (6 points)

   (a) Find the tangent plane approximation $L(x, y)$ of the function

   $$f(x, y) = \sin(2x + 3y) + 1$$

   at the point $(-3, 2)$.

   (b) Use the approximation above to estimate the value of $f(-2.8, 2.3)$.

(a) $L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$\nabla f = \langle 2\cos(2x+3y), 3\cos(2x+3y)\rangle$

$\Rightarrow \nabla f(-3,2) = \langle 2, 3\rangle$

and $f(-3,2) = 1$.

So $L(x,y) = 1 + 2(x+3) + 3(y-2) = 2x+3y+1$.

(b) $f(x,y) \approx L(x,y)$

$= L(-2.8, 2.3)$

$= 1 + (2)(-2.8+3) + (3)(2.3-2)$

$= 1 + (2)(+0.2) + (3)(0.3)$

$= 1 + 0.4 + 0.9$

$\Rightarrow f(-2.8, 2.3) \approx 2.3$

#	Score
1	
2	
3	
4	
Σ	

The part related to critical points can be in our exam.

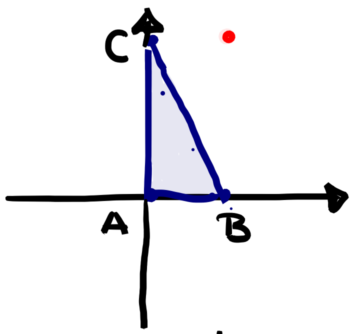
2. (6 points) Find the absolute maximum and absolute minimum of

$$f(x,y) = 2 + xy - 2x - \frac{1}{4}y^2$$

in the closed triangular region with vertices given by $(0,0)$, $(1,0)$, and $(0,2)$. Justify your answer.

$\nabla f = \langle y-2, x - \frac{y}{2} \rangle = \langle 0,0 \rangle$

$\Rightarrow \begin{cases} y = 2 \\ x = \frac{y}{2} \end{cases} \Rightarrow$ it has only one critical pt $(1,2)$



So its only critical point is NOT in the region in which we are interested. So we should examine the boundary:

Segment AB $\quad y=0,\ 0 \le x \le 1$

$f(x,0) = 2-2x$ which is a decreasing function. So its max on AB is $\boxed{f(0,0)=2}$, and its min on AB is $\boxed{f(1,0)=0}$.

Segment AC $\quad x=0,\ 0 \le y \le 2$

$g(y) = f(0,y) = 2 - \frac{1}{4}y^2 \Rightarrow g'(y) = \frac{-y}{2} = 0$

$\Rightarrow$

y	0	2
$g'(y)$	0	$-$
$g(y)$	2	$\searrow$ 0

. So the max of f on AC is $\boxed{f(0,0)=2}$ and its min on AC is $\boxed{f(0,2)=1}$

Segment BC $\quad t\overrightarrow{OB} + (1-t)\overrightarrow{OC} = t\langle 1,0\rangle + (1-t)\langle 0,2\rangle$

$= \langle t, 2-2t \rangle$ for $0 \le t \le 1$

$h(t) = f(t, 2-2t) = 2 + t(2-2t) - 2t - \frac{1}{4}(2-2t)^2$

$$= 2 + 2t - 2t^2 - 2t - \frac{(4 - 8t + 4t^2)}{4}$$

$$= 2 - 2t^2 - 1 + 2t - t^2$$

$$= 1 + 2t - 3t^2$$

$$\Rightarrow h'(t) = 2 - 6t = 0 \quad \Rightarrow t = \frac{1}{3}$$

t	0		$1/3$		1
$h'(t)$	2	$+$	0	$-$	-4
$h(t)$	1	$\nearrow$	$\frac{4}{3}$	$\searrow$	0

So $\boxed{f(\frac{1}{3}, \frac{4}{3}) = \frac{4}{3}}$ is the max of f on BC

and $\boxed{f(1, 0) = 0}$ is the min of f on BC

Hence by comparing the above values we have that

$f(0,0) = 2$ is the absolute max.

and $f(1,0) = 0$ is the absolute min.

NOT PART OF OUR EXAM

3. (6 points) Using Lagrange multipliers find the maximum and minimum values of

$$f(x,y) = 2(x+1)y,$$

subject to the constraint

$$x^2 + y^2 = 1.$$

Show all your work.

We have to solve the following system of equations:

$$\begin{cases} \nabla f = c\nabla g \\ g = 1 \end{cases} \Rightarrow \begin{cases} \langle 2y, 2x+2\rangle = c\langle 2x, 2y\rangle \\ x^2+y^2=1 \end{cases}$$

$$\Rightarrow \begin{cases} y = cx \\ x+1 = cy \\ x^2+y^2 = 1 \end{cases} \Rightarrow \begin{cases} x+1 = c^2x \\ x^2+c^2x^2 = 1 \end{cases} \Rightarrow \begin{cases} x^2+x = c^2x^2 \\ 1-x^2 = c^2x^2 \end{cases}$$

$$\Rightarrow x^2+x = 1-x^2 \Rightarrow 2x^2+x-1 = 0$$

$$\Rightarrow (2x-1)(x+1) = 0 \Rightarrow x = \frac{1}{2} \text{ or } x = -1$$

$x = \frac{1}{2} \Rightarrow y = \frac{\pm\sqrt{3}}{2}$ as $x^2+y^2 = 1$.

$x = -1 \Rightarrow y = 0$ as $x^2+y^2=1$.

$$f\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = 2\left(\frac{3}{2}\right)\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}.$$

$$f\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right) = 2\left(\frac{3}{2}\right)\left(\frac{-\sqrt{3}}{2}\right) = -\frac{3\sqrt{3}}{2}.$$

$$f(-1, 0) = 2(0)(0) = 0$$

So $3\sqrt{3}/2$ is the max and $-3\sqrt{3}/2$ is the min.

NOT PART OF OUR EXAM

4. (6 points) Compute the double integral of the function

$$f(x, y) = \frac{x}{y} e^{3x^2},$$

in the domain

$$R = \{(x, y) \in I\!R^2 : \quad 0 \le x \le 1, \quad 1 \le y \le 3\}.$$

Show all your work.

$$\int_0^1 \int_1^3 \frac{x}{y} e^{3x^2} \, dy \, dx$$

$$= \int_0^1 x e^{3x^2} \ln(y) \Big|_{y=1}^{y=3} dx$$

$$= \int_0^1 \ln(3) \, x e^{3x^2} dx$$

Let $u = 3x^2 \Rightarrow du = 6x\,dx$

$$\Rightarrow x e^{3x^2} dx = \frac{1}{6} e^u du.$$

$$= \int_0^3 \ln(3) \frac{1}{6} e^u du$$

$$= \frac{\ln(3)}{6} e^u \Big|_0^3 = \frac{\ln(3)}{6}(e^3 - 1).$$