1. Write your Name, PID, and Section on the front of your Blue Book.
2. Write the Version of your exam on the front of your Blue Book.
3. No calculators or other electronic devices are allowed during this exam.
4. You may use one page of notes, but no books or other assistance during this exam.
5. Read each question carefully, and answer each question completely.
6. Write your solutions clearly in your Blue Book
   (a) Carefully indicate the number and letter of each question.
   (b) Present your answers in the same order they appear in the exam.
   (c) Start each question on a new page.
7. Show all of your work; no credit will be given for unsupported answers.

1. (a) (8 points) Find the area of the triangle PQR where $P = (1, 1, 1)$, $Q = (1, 2, 3)$ and $R = (0, 1, 1)$.
   (b) (4 points) Find an equation of the plane which passes through P, Q, and R.

2. The velocity vector of a moving particle at time t is

$$\mathbf{v}(t) = \langle 2t\cos(t^2), -2t\sin(t^2), t^2 - 1\rangle.$$

   (a) (2 points) Find the particle's acceleration $\mathbf{a}(t)$ as a function of t.
   (b) (4 points) Find the particle's position vector $\mathbf{r}(t)$ as a function of t if the initial position $\mathbf{r}(0) = \langle 1, 0, 1\rangle$.
   (c) (4 points) Find the total distance traveled by the particle during the time interval $1 \leq t \leq 2$.

3. (5 points) Suppose that $z = f(x, y)$ satisfies $xe^z + ze^y = x + y$. Calculate $\partial z/\partial x$ as a function of x, y, and z.

4. (5 points) Find all the points on the ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 1$, where the tangent plane is parallel to $z = 1 - x - y$.

5. (a) (8 points) Let $f(x, y) = \sqrt{x^2 + y^2}$. Find an equation of the tangent plane of the graph $z = f(x, y)$ at $(3, 4, 5)$.
   (b) (4 points) Use a linear approximation to estimate $f(3.1, 4.2)$.

6. (a) (8 points) Find the directional derivative of $f(x, y, z) = zx^2 - y^2$ at $P = (1, 1, 1)$ in the direction of $\mathbf{v} = \langle -1, 1, 1\rangle$. Explain if the function increases or decreases?
   (b) (2 points) Find the maximum rate of increase of f at $P = (1, 1, 1)$.

7. (12 points) Find the critical points of f, and determine local minimum, local maximum, and the saddle points, where $f(x, y) = x^4 - 4xy + 2y^2$.

8. (a) (2 points) Explain why $f(x, y) = x^2y + 1$ has a maximum and a minimum on the ellipse $4x^2 + 9y^2 = 36$.
   (b) (8 points) Find the maximum and the minimum of f subject to the constraint $4x^2 + 9y^2 = 36$.

9. (a) (4 points) Sketch the domain of integration

$$\int_0^1 \int_{x^2}^{x} \frac{\sin(\pi y)}{\sqrt{y} - y} dy dx.$$

(b) (4 points) Change the order of integration.

(c) (4 points) Evaluate the integral.

10. (12 points) Find the volume of the solid enclosed by the xy-plane, the paraboloid $z = 4 - x^2 - y^2$, and the cylinder $x^2 + y^2 = 1$.

Good luck.