Math 103A: Winter 2014
Homework 1
Due 5:00pm on Friday 1/10/2014

Problem 1: (Exercise 0.4 in Gallian) Find integers s and t such that $1 = 7s + 11t$. Show that s and t are not unique.

Problem 2: (Exercise 0.5 in Gallian) Show that if a and b are positive integers, we have that $ab = \text{lcm}(a, b) \cdot \gcd(a, b)$.

Problem 3: (Exercise 0.9 in Gallian) Let n be a fixed integer greater than 1. If a mod $n = a'$ and b mod $n = b'$, prove that $(a + b) \bmod n = (a' + b') \bmod n$ and ab mod $n = a'b' \bmod n$.

Problem 4: (Exercise 0.11 in Gallian) Let n and a be positive integers and let $d = \gcd(a, n)$. Show that the equation $ax \bmod n = 1$ has a solution if and only if $d = 1$.

Problem 5: (Exercise 0.16 in Gallian) Determine 7^{1000} mod 6 and 6^{1001} mod 7.

Problem 6: Let $a, b, s,$ and t be integers. If a mod $st = b$ mod st, show that a mod s $= b$ mod s and a mod $t = b$ mod t.

Problem 7: (Exercise 0.58 in Gallian) Let S be the set of real numbers. For $a, b \in S$, define $a \sim b$ if $a - b$ is an integer. Show that $\sim$ is an equivalence relation on S and describe the equivalence classes of S.

Problem 8: (Exercise 0.59 in Gallian) Let S be the set of integers. For $a, b \in S$, define aRb if $ab \geq 0$. Is R an equivalence relation on S?

Problem 9: (Exercise 0.60 in Gallian) Let S be the set of integers. For $a, b \in S$, define aRb if $a + b$ is even. Prove that R is an equivalence relation on S and determine the equivalence classes of S.