Math 103A: Winter 2014
Homework 2
Due 5:00pm on Friday 1/17/2014

Problem 1: (Exercise 1.14 in Gallian) Describe the symmetries of a parallelogram that is neither a rectangle nor a rhombus. Describe the symmetries of a rhombus that is not a rectangle.

Problem 2: (Exercise 1.16 in Gallian) Consider an infinitely long strip of equally spaced H's:

$$\cdots \text{HHHH} \cdots$$

Describe the symmetries of the strip. Is the group of symmetries of the strip Abelian?

Problem 3: (Exercise 2.2 in Gallian) Which of the following binary operations are associative?

(1) multiplication mod n
(2) division of nonzero rational numbers
(3) function composition of polynomials with real coefficients
(4) multiplication of 2×2 matrices with integer entries

Problem 4: (Exercise 2.4 in Gallian) Which of the following sets are closed under the given operation?

(1) $\{0, 4, 8, 12\}$ under addition mod 16
(2) $\{0, 4, 8, 12\}$ under addition mod 15
(3) $\{1, 4, 7, 13\}$ under multiplication mod 15
(4) $\{1, 4, 5, 7\}$ under multiplication mod 9

Problem 5: (Exercise 2.10 in Gallian) Let $GL(2, \mathbb{R})$ be the group of 2×2 invertible matrices with real entries. Prove that $GL(2, \mathbb{R})$ is not Abelian.

Problem 6: Define $GL(3, \mathbb{R})_+$ to be the set of all 3×3 real matrices A such that $\det(A) > 0$. Prove that $GL(3, \mathbb{R})_+$ is a group under matrix multiplication.

Problem 7: (Exercise 2.26 in Gallian) Let G be a group and let $a \in G$. Prove that $(a^{-1})^{-1} = a$.

Problem 8: Let G be a group. Prove that G is Abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$.

Problem 9: Let G be a group, let $a, b \in G$, and let n be a positive integers. Prove that $(aba^{-1})^n = ab^na^{-1}$.