Math 103A: Winter 2014
Homework 3
Due 5:00pm on Friday 1/24/2014

Problem 1: (Exercise 2.26 in Gallian) Let G be a group with the property that for any $x, y, z \in G$, $xy = zx$ implies $y = z$. Prove G is Abelian.

Problem 2: (Exercise 2.32 in Gallian) Construct a Cayley table for $U(12)$.

Problem 3: (Exercise 2.46 in Gallian) Prove that the set of all rational numbers of the form $2^m 3^n$, where m and n are integers, is a group under multiplication.

Problem 4: (Exercise 2.48 in Gallian) Prove that the set of 3×3 real matrices of the form $\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$ (with $a, b, c \in \mathbb{R}$) forms a group under matrix multiplication.

Problem 5: Let G be a finite group of even order. Prove that there exists a non-identity element $x \in G$ with $x^2 = e$.

Problem 6: (Exercise 3.4 in Gallian) Let G be a group and let $g \in G$. Prove that g and g^{-1} have the same order.

Problem 7: (Exercise 3.10 in Gallian) How many subgroups of order 4 does D_4 have?

Problem 8: Let G be an Abelian group and let $g, h \in G$. Prove that if g and h have finite order, then gh has finite order.

Problem 9: Give an example two matrices $A, B \in GL(2, \mathbb{R})$ such that $|A| = |B| = 2$, but AB has infinite order.