Math 103A: Winter 2014
Homework 4
Due 5:00pm on Friday 2/7/2014

Problem 1: Let G be a group and let $\{H_i\}_{i\in I}$ be a collection of subgroups of G (where I is some nonempty index set). Prove that the intersection $\cap_{i\in I} H_i$ is a subgroup of G. (Be careful not to assume that I is finite or countable!)

Problem 2: List all the cyclic subgroups of $U(15)$.

Problem 3: (3.80 in Gallian) Let G be a finite group with more than one element. Show that G contains an element of prime order.

Problem 4: Let $p > 0$ be a prime number and let G be a cyclic group. Suppose that G has exactly three subgroups: G itself, $\{e\}$, and a subgroup of order p. What is $|G|$?

Problem 5: (Exercise 4.22 in Gallian) Prove that every group of order 3 is cyclic.

Problem 6: (Exercise 4.32 in Gallian) Determine the subgroup lattice for Z_{12}.

Problem 7: (Exercise 4.66 in Gallian) Prove that $U(2^n)$ is not cyclic for $n \geq 3$.

Problem 8: (Exercise 5.2 in Gallian) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{pmatrix}$. Write α, β, and $\alpha\beta$ as (a) products of disjoint cycles and (b) products of 2-cycles.

Problem 9: (Exercise 5.28 in Gallian) How many elements of order 5 are there in S_7?