Math 103A: Winter 2014
Homework 5
Due 5:00pm on Friday 2/14/2014

Problem 1: (Exercise 5.26 in Gallian) Let $\alpha, \beta \in S_n$. Prove that $\alpha\beta\alpha^{-1}\beta^{-1}$ is an even permutation.

Problem 2: (Exercise 5.32 in Gallian) Let $\beta = (123)(145)$. Write β^{99} in disjoint cycle form.

Problem 3: (Exercise 5.46 in Gallian) Prove that A_n is non-Abelian for all $n \geq 4$.

Problem 4: (Exercise 5.78 in Gallian) Find five subgroups of S_5 of order 24.

Problem 5: (Exercise 6.6 in Gallian) Prove that group isomorphism gives an equivalence relation. That is, prove that for any groups G, H, and K, we have $G \approx G$, $G \approx H$ if and only if $H \approx G$, and if $G \approx H$ and $H \approx K$ implies $G \approx K$.

Problem 6: (Exercise 6.10 in Gallian) Let G be a group. Prove that the function $\alpha : G \to G$ given by $\alpha(g) = g^{-1}$ is an automorphism of G if and only if G is Abelian.

Problem 7: (Exercise 6.34 in Gallian) Prove or disprove that $U(20)$ and $U(24)$ are isomorphic.

Problem 8: Let G be a finite Abelian group containing no element of order 2. Let $\psi : G \to G$ be the mapping $\psi(g) = g^2$. Prove that $\psi \in \text{Aut}(G)$.

Problem 9: Let G be a group and let $\beta \in \text{Aut}(G)$. Prove that $\{g \in G : \beta^2(g) = g\}$ is a subgroup of G.