Math 103A: Winter 2014
Homework 6
Due 5:00pm on Friday 2/21/2014

Problem 1: (Exercise 7.8 in Gallian) Suppose that a has order 15. Find all of the left cosets of $\langle a^5 \rangle$ in $\langle a \rangle$.

Problem 2: (Exercise 7.10 in Gallian) Find an example of a group G and subgroups H and K such that $HK = \{hk : h \in H, k \in K\}$ is not a subgroup of G.

Problem 3: Suppose H and K are subgroups of a group G. If $|H| = 12$ and $|K| = 35$, what is $|H \cap K|$?

Problem 4: Let $G = GL(n, \mathbb{R})$ and let $H = SL(n, \mathbb{R})$. Let A be an $n \times n$ real matrix such that $\det(A) = 3$. Prove that

$$AH = \{B \in \mathrm{Mat}_{n \times n}(\mathbb{R}) : \det(B) = 3\}.$$

Problem 5: (Exercise 7.40 in Gallian) Prove that any group of order 63 contains an element of order 3.

Problem 6: (Exercise 7.64 in Gallian) A soccer ball has 20 faces that are regular hexagons and 12 faces that are regular pentagons. Use the Orbit-Stabilizer Theorem to show that a soccer ball cannot have a 60° rotational symmetry about a line through the centers of two opposite hexagonal faces. (Hint: You may use without proof the fact that the group of rotational symmetries of a soccer ball has order 60 – and is in fact isomorphic to A_5.)

Problem 7: (Exercise 8.12 in Gallian) Give examples of four groups of order 12, no two of which are isomorphic. Give reasons why no two are isomorphic.

Problem 8: (Exercise 8.16 in Gallian) Suppose that $G_1 \approx G_2$ and $H_1 \approx H_2$. Prove that $G_1 \oplus H_1 \approx G_2 \oplus H_2$.

Problem 9: (Exercise 8.30 in Gallian) Find all subgroups of order 4 in $Z_4 \oplus Z_4$.