Math 103A: Winter 2014
Homework 8
Due 5:00pm on Friday 3/14/2014

Problem 1: (Exercise 10.28 in Gallian) Suppose $\phi : S_4 \to Z_2$ is a surjective homomorphism. Determine $\mathrm{Ker}(\phi)$. Determine *all* homomorphisms $S_4 \to Z_2$.

Problem 2: (Exercise 10.54 in Gallian) Let G and H be groups and let $\alpha, \beta : G \to H$ be homomorphisms. Let $K = \{g \in G : \alpha(g) = \beta(g)\}$. Prove or disprove that K is necessarily a subgroup of G.

Problem 3: (Exercise 10.58 in Gallian) Suppose H and K are normal subgroups of G and that $H \cap K = \{e\}$. Prove that G is isomorphic to a subgroup of $G/H \oplus G/K$.

Problem 4: (Exercise 10.66 in Gallian) Let p be a prime. Determine the number of homomorphisms $Z_p \oplus Z_p \to Z_p$.

Problem 5: (Exercise 10.30 in Gallian) Suppose $\phi : G \to Z_6 \oplus Z_2$ is a surjective homomorphism and the kernel of ϕ has order 5. Show that G has normal subgroups of orders $5, 10, 15, 20, 30$, and 60.

Problem 6: (Exercise 11.4 in Gallian) Find the number of elements of order 2 in each of $Z_{16}, Z_8 \oplus Z_2$, and $Z_4 \oplus Z_2 \oplus Z_2$. Do the same for elements of order 4.

Problem 7: (Exercise 11.18 in Gallian) Let $p_1, p_2, \ldots, p_n$ be distinct primes. Up to isomorphism, how many Abelian groups are there of order $p_1^4 p_2^4 \cdots p_n^4$?

Problem 8: (Exercise 11.28 in Gallian) The set $G = \{1, 4, 11, 14, 16, 19, 26, 29, 31, 34, 41, 44\}$ is a group under multiplication mod 45. Write G as an external and an internal direct product of cyclic groups of prime-power order.

Problem 9: (Exercise 11.34 in Gallian) Let G be the group of all $n \times n$ diagonal matrices with ± 1 diagonal entries. What is the isomorphism class of G?