Math 103A: Winter 2014
Midterm 1

Instructions: Please write your name on your blue book. Make it clear in your blue book what problem you are working on. Write legibly and justify your answers. This exam is graded out of 100 points. Following these instructions is worth 5 points.

Problem 1: [5 + 10 pts.] (a) Carefully define the "special linear group" $SL(2, \mathbb{R})$. (b) Is $SL(2, \mathbb{R})$ Abelian? (Be sure to justify your answer.)

Problem 2: [15 pts.] Recall that Z denotes the set of integers. Define a relation $\sim$ on Z by $a \sim b$ if and only if $ab \geq 0$. Is $\sim$ an equivalence relation on Z? (Be sure to justify your answer.)

Problem 3: [15 pts.] Recall that $U(10)$ denotes the group $\{1 \leq i \leq 10 : \gcd(i, 10) = 1\}$ under multiplication mod 10. Draw the Cayley table (i.e., the multiplication table) of $U(10)$.

Problem 4: [15 pts.] Is the set $\mathrm{Mat}_{2\times 2}(\mathbb{R})$ of 2×2 real matrices a group under matrix multiplication? (Be sure to justify your answer.)

Problem 5: [20 pts.] Give an example of a group G and a subgroup $H < G$ such that G has infinite order and H has order six.

Problem 6: [15 pts.] Prove or give a counterexample: Every finite group G of order n contains an element $g \in G$ of order n.