Math 103A: Winter 2014
Practice Midterm 1

Instructions: Please write your name on your blue book. Make it clear in your blue book what problem you are working on. Write legibly and justify your answers. This exam is graded out of 100 points. Following these instructions is worth 5 points.

Problem 1: [5 + 10 pts.] Let S be a set and let $\star$ be a binary relation on S. (a) Carefully define what it means for $\star$ to be "associative" and (b) give an example of a set S and a binary operation $\star$ on S which is not associative. (Be sure to justify your answer.)

Problem 2: [20 pts.] Give an example, with justification, of two matrices $A, B \in GL(2, \mathbb{R})$ such that $|A| = |B| = 2$, but AB has infinite order.

Problem 3: [15 pts.] True or false: For any two positive integers a, b satisfying $\gcd(a, b) = 1$, there exist unique integers s, t such that $as + bt = 1$. (Be sure to justify your answer.)

Problem 4: [15 pts.] Compute 2013^{2015} mod 2014.

Problem 5: [15 pts.] Give an example of a group G and a subgroup $H < G$ such that G has order eight and H has order four.

Problem 6: [15 pts.] Recall that an $n \times n$ real matrix A is *orthogonal* if $AA^T = I$. Let $O(n, \mathbb{R})$ denote the set of all $n \times n$ real orthogonal matrices. Prove that $O(n, \mathbb{R})$ is a subgroup of $GL(n, \mathbb{R})$. ($O(n, \mathbb{R})$ is called the *orthogonal group*.)