Math 103A: Winter 2014
Practice Midterm 1 Solutions

Problem 1: (a) $\star$ is associative if $(a \star b) \star c = a \star (b \star c)$ for all $a, b, c \in S$.
(b) Let $S = Z$ and define $\star$ by $a \star b = a - b$. Then $(1 \star 1) \star 1 = (1 - 1) - 1 = 0 - 1 = -1$, but $1 \star (1 \star 1) = 1 - (1 - 1) = 1 - 0 = 1$.

Problem 2: (See the solution to Problem 9 on Homework 3.)

Problem 3: This is false. Indeed, if $a = 2$ and $b = 3$, we have that $(s, t) = (-1, 1)$ and $(s, t) = (-4, 3)$ are both solutions to $as + bt = 1$.

Problem 4: We know that 2013 mod 2014 $= -1$ mod 2014, so 2013^{2015} mod 2014 $= (-1)^{2015}$ mod 2014 $= -1$ mod 2014 $= 2013$.

Problem 5: Let $G = D_4$ be the dihedral group of symmetries of the square. Let $H = \{R_0, R_{90}, R_{180}, R_{270}\}$ be the subgroup of G consisting of the rotations.

Problem 6: Since $II^T = I$, we have that $I \in O(n, \mathbb{R})$ and $O(n, \mathbb{R})$ is nonempty. Suppose $A, B \in O(n, \mathbb{R})$. Then B is invertible with $B^{-1} = B^T$. Also, we have that $(AB^{-1})(AB^{-1})^T = A(B^{-1}(B^{-1})^T)A^T = A(B^{-1}B)A^T = AA^T = I$. We conclude that $AB^{-1} \in O(n, \mathbb{R})$, so that $O(n, \mathbb{R})$ is a subgroup of $GL(n, \mathbb{R})$, as desired.