Math 103A: Winter 2014
Midterm 2

Instructions: Please write your name on your blue book. Make it clear in your blue book what problem you are working on. Write legibly and explain your reasoning. This exam is graded out of 100 points. Following these instructions is worth 5 points.

Problem 1: [15 points] (a) Carefully define what it means for a group G to be "cyclic". (b) Prove or give a counterexample: If G and H are cyclic groups, then $G \oplus H$ is also a cyclic group.

Problem 2: [15 points] Let G be a group of order 63. Prove that G contains an element of order 3.

Problem 3: [15 points] Let S be the square in the plane $\mathbb{R}^2$ which is centered at the origin and has side length 4. Let $P = (0, 1) \in \mathbb{R}^2$. Let the dihedral group D_4 act on S by permutations. (a) What is the orbit $\mathrm{orb}_{D_4}(P)$? (b) What is the stabilizer $\mathrm{stab}_{D_4}(P)$?

Problem 4: [15 points] Let $\alpha = (2, 1, 4, 5, 6, 3) \in S_6$ and let $\beta = (1, 3, 2)(5, 6) \in S_6$. Express the product $\alpha\beta$ as a product of disjoint cycles and determine the order $|\alpha\beta|$.

Problem 5: [20 points] Give an example of a group G and an automorphism $\phi \in \mathrm{Aut}(G)$ such that $\phi \notin \mathrm{Inn}(G)$.

Problem 6: [15 points] Let G be the group of 2×2 real matrices of the form $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$. Prove that $G \approx \mathbb{R}$.