

Exam 2, Mathematics 109

Dr. Cristian D. Popescu

November 14, 2005

Name:

Student ID:

Section Number:

Note: There are 3 problems on this exam. You will not receive credit unless you show all your work. No books, calculators, notes or tables are permitted.

I. (40 points) For each natural number n , let

$$A_n = \left(-\frac{2}{n}, \frac{3n-1}{n} \right).$$

Find $\bigcup_{n \in \mathbb{N}} A_n$ and $\bigcap_{n \in \mathbb{N}} A_n$. Justify your answers.

II. (30 points) Let $(a_n)_{n \in \mathbb{N}}$ be the sequence defined recursively as follows

$$a_1 = a_2 = 1, \quad a_{n+1} = \frac{1}{2} \left(a_n + \frac{2}{a_{n-1}} \right), \quad \forall n \geq 2.$$

Prove that for one has $1 \leq a_n \leq 2$, for all natural numbers n .

III. (30 points)

- (1) Find the set of all integral solutions for the following Diophantine equation.

$$200 \cdot x + 35 \cdot y = 15.$$

- (2) Let (x, y) and (x', y') be two solutions for the Diophantine equation in (1) above. Show that $2 \mid (y - y')$ and $7 \mid (x - x')$.