Problem 1.1: *State which of the following are propositions.*

a) proposition
b) This is simply a mathematical expression. Nothing is being asserted here.
c) proposition
d) No question can be a propostion.
e) "I" acts here as a variable, so no, this is not a proposition.

Problem 1.4: *Assume that "Mary is a girl" is a true statement and that "Mary is ten years old" is a true statement. Which of the following are true?*

a) *If Mary is ten years old, then Mary is a girl.*
The conclusion "Mary is a girl" is true, and thus the conditional proposition is true.

b) *Mary is ten years old iff Mary is a girl.*
Both statements are true, and thus the proposition is true.

Problem 1.7: *Write the converse and the contrapositive of the following propositions.*

a) *If* $\sqrt{2} < \sqrt{5}$*, then* $2 < 5$.
Converse: If $2 < 5$, then $\sqrt{2} < \sqrt{5}$.
Contrapositive: If $2 \geq 5$, then $\sqrt{2} \geq \sqrt{5}$.

b) *If* $2 \geq 5$*, then* $\sqrt{2} \geq \sqrt{5}$.
Converse: If $\sqrt{2} \geq \sqrt{5}$, then $2 \geq 5$.
Contrapositive: If $\sqrt{2} < \sqrt{5}$, then $2 < 5$.

Problem 1.8:

	First Guy	Second Guy	Third Guy
1	T	T	T
2	T	T	F
3	T	F	T
4	T	F	F
5	F	T	T
6	F	T	F
7	F	F	T
8	F	F	F

The first guy can't be telling the truth, so we may immediately eliminate the first four possibilities and the last. If the second guy were lying, then there's only one liar and the seventh

possibility cannot hold. Thus, the second guy has to be telling the truth.

Problem 1.11:
Question: As a Texan, would you like to see a Texan president or a Texan vice-president?
Answer: Yes.
Note that the reporter did not use an "exclusive or" – see Problem 1.12.

Problem 1.12: The connective $\oplus$ is called *exclusive or* because $P \oplus Q$ is true when only one of P and Q is true.

Problem 1.15: *Prepare a truth table for each of the following expressions.*

a) $P \wedge Q \rightarrow P \vee Q$

P	Q	$P \wedge Q$	$P \vee Q$	$P \wedge Q \rightarrow P \vee Q$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

b) $P \rightarrow (Q \rightarrow R)$

P	Q	R	$Q \rightarrow R$	$P \rightarrow (Q \rightarrow R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

Problem 1.18: **c)** $(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$

P	Q	$(P \rightarrow Q)$	$\neg Q$	$\neg P$	$(\neg Q \rightarrow \neg P)$	$(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

d) $P \rightarrow (Q \rightarrow R)$
Same as Problem 1.15(b).

f) $\neg(P \vee Q) \rightarrow R$

P	Q	R	$P \vee Q$	$\neg(P \vee Q)$	$\neg(P \vee Q) \rightarrow R$
T	T	T	T	F	T
T	T	F	T	F	T
T	F	T	T	F	T
T	F	F	T	F	T
F	T	T	T	F	T
F	T	F	T	F	T
F	F	T	F	T	T
F	F	F	F	T	F

Problem 1.22: *Which of the following statements are true?*

a) $(P \leftrightarrow Q) \Leftrightarrow (Q \leftrightarrow P)$

P	Q	$P \leftrightarrow Q$	$Q \leftrightarrow P$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

Thus $P \leftrightarrow Q$ and $Q \leftrightarrow P$ have the same truth values, and the statement is true.

b) $\neg(P \leftrightarrow Q) \Leftrightarrow (\neg P \rightarrow \neg Q)$

P	Q	$\neg P$	$\neg Q$	$\neg(P \leftrightarrow Q)$	$\neg P \rightarrow \neg Q$
T	T	F	F	F	T

The statement is false.

c) $(\neg P \leftrightarrow Q) \Leftrightarrow (P \leftrightarrow \neg Q)$

P	Q	$\neg P$	$\neg Q$	$\neg P \leftrightarrow Q$	$P \leftrightarrow \neg Q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

Thus $\neg P \leftrightarrow Q$ and $P \leftrightarrow \neg Q$ have the same truth values and the statement is true.

d) $(\neg P \rightarrow Q) \Leftrightarrow (\neg P \wedge \neg Q)$

P	Q	$\neg P$	$\neg Q$	$\neg P \rightarrow Q$	$\neg P \wedge \neg Q$
T	T	F	F	T	F

The statement is false.

Problem 1.25: *Write, in symbols, the converse, the contrapositive, and the negation of each of the following propositional expressions.*

a) $P \rightarrow (Q \vee R)$

Converse: $(Q \vee R) \rightarrow P$
Contrapositive: $\neg(Q \vee R) \rightarrow \neg P$ or $(\neg Q \wedge \neg R) \rightarrow \neg P$ (either is acceptable)
Negation: $\neg(P \rightarrow (Q \vee R)) \leftrightarrow P \wedge \neg(Q \vee R)$ which can be written also as $P \wedge \neg Q \wedge \neg R$. (This is the best possible answer.)

b) $P \rightarrow (Q \wedge R)$

Converse: $(Q \wedge R) \rightarrow P$
Contrapositive: $\neg(Q \wedge R) \rightarrow \neg P$ or $(\neg Q \vee \neg R) \rightarrow \neg P$
Negation: As before, $P \wedge (\neg Q \vee \neg R)$.

Problem 1.28: *By constructing truth tables, prove each of the following is a tautology.*
You must show that the right and left hand sides have the same truth values.

a) *Proposition 1.1(a):* $(P \vee Q) \leftrightarrow (\neg P \rightarrow Q)$

P	Q	$P \vee Q$	$\neg P$	$\neg P \rightarrow Q$	$(P \vee Q) \leftrightarrow (\neg P \rightarrow Q)$
T	T	T	F	T	T
T	F	T	F	T	T
F	T	T	T	T	T
F	F	F	T	F	T

b) *Proposition 1.1(b):* $\neg(P \to Q) \leftrightarrow (P \wedge \neg Q)$

P	Q	$P \to Q$	$\neg(P \to Q)$	$\neg Q$	$P \wedge \neg Q$	$\neg(P \to Q) \leftrightarrow (P \wedge \neg Q)$
T	T	T	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	F	T
F	F	T	F	T	F	T

c) *Proposition 1.1(c):* $\neg(P \vee Q) \leftrightarrow (\neg P \wedge \neg Q)$

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$\neg(P \vee Q) \leftrightarrow (\neg P \wedge \neg Q)$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T