Exam 1, Mathematics 20C
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Name:
Student ID:
Section Number:

Note: There are 3 problems on this exam. You will not receive credit unless you show all your work. No books, calculators, notes or tables are permitted.

I. (30 points)

(1) Sketch the curve with polar equation $r = 2\cos\theta$.
(2) Write the equation of the tangent line to the curve in (1) at the point corresponding to $\theta = \frac{\pi}{4}$.
(3) Find the points on the curve above where the tangent to the curve is vertical and horizontal, respectively.
(4) Find a cartesian equation for the curve in (1).

II. (40 points) Let $\vec{a} =< 0,\ 1,\ 1 >$ and $\vec{b} =< 1,\ 1,\ 0 >$.

(1) Compute the angle $\theta \in [0, \pi)$ determined by the vectors $\vec{a}$ and $\vec{b}$.
(2) Find a unit vector $\vec{u}$, which is perpendicular on both $\vec{a}$ and $\vec{b}$.
(3) Find the volume of the parallelepiped determined by the vectors $\vec{a}$, $\vec{b}$, $\vec{u}$.
(4) Write the equation of the plane passing through $P_0(0, 0, 0)$ and which is perpendicular on $\vec{a}$.

III. (40 points) Let $(\pi_1) : x + y + z - 1 = 0$ and $(\pi_2) : 2x + 2y - 2z = 0$ be the equations of two planes (π_1) and (π_2).

(1) Find the angle $\theta \in [0, \pi)$ determined by the two planes above.
(2) Find the vectorial, parametric and symmetric equations of the line of intersection between the two planes above.
(3) Find the distance between the point $P_0(1, 1, 1)$ and the plane (π_1).
(4) Write the cartesian equation of the sphere centered at $P_0(1, 1, 1)$ and tangent to (π_1).
(5) Find the point of intersection between the plane (π_1) and the line which passes through $P_0(1, 1, 1)$ and is perpendicular on (π_1).