Exam 2, Mathematics 20C
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Name:
Student ID:
Section Number:

Note: There are 3 problems on this exam. You will not receive credit unless you show all your work. No books, calculators, notes or tables are permitted.

I. (35 points) Let $\mathcal{C}$ be the space curve whose vectorial equation is given by

$$\vec{r}(t) = t\,\vec{i} + t^2\,\vec{j} + \frac{2t^3}{3}\,\vec{k}, \qquad t \in \mathbb{R}.$$

(1) Find the length of the arc of $\mathcal{C}$ corresponding to $0 \leq t \leq 1$.
(2) Find the unit tangent vector $\vec{T}$, unit normal vector $\vec{N}$, and unit binormal vector $\vec{B}$ to the curve $\mathcal{C}$ at the point $P(0,0,0)$.
(3) Find the equation of the osculating plane to the curve $\mathcal{C}$ at the point $P(0,0,0)$.
(4) Find the curvature κ of the curve $\mathcal{C}$ at $P(0,0,0)$.
(5) What is the radius of the osculating circle to the curve $\mathcal{C}$ at $P(0,0,0)$?

II. (30 points) Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be the function given by

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0)\,. \end{cases}$$

(1) Compute $\lim_{(x,y)\to(0,0)} f(x,y)$ and determine whether the function f is continuous at $(0,0)$ or not.

(2) Compute $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$ and determine whether the function f is differentiable at $(0,0)$ or not.

III. (35 points) Let $f : \mathbb{R}^3 \longrightarrow \mathbb{R}$ be the function given by

$$f(x, y, z) = x^3 + y^3 + z^3 + xyz\,.$$

(1) Assuming that x, y, and z are functions of t given by $x(t) = t$, $y(t) = \sin t$, and $z(t) = \cos t$, compute df/dt at $t = 0$.
(2) Compute the gradient ∇f at $P(1, 1, 1)$.
(3) Determine the directional derivative of f at $P(1, 1, 1)$ in the direction of $\vec{v} = 3\vec{\imath} + 4\vec{\jmath}$. **(Please note that $\vec{v}$ is not a unit vector.)**
(4) Write down the equation of the plane tangent to the surface $f(x, y, z) = 4$ at $P(1, 1, 1)$.