September 12, 2006

Course Announcement – Math 205A-B, Fall 2006 -Winter 2007

LOCAL AND GLOBAL CLASS FIELD THEORY

Local class field theory is the precise description of the maximal abelian extension of an arbitrary **local field.** A field is called local if it is complete with respect to a discrete valuation and has a finite residue field. It turns out that a field is local if and only if it is either a finite extension of a ***p***-adic field ***Qp*** or a field of power series in one variable with coefficients in a finite field ***Fq((X)).*** Consequently, local fields arise naturally as completions of **global fields** (i.e. number fields or function fields in one variable over finite fields) with respect to their non-Archimedean (discrete) valuations. There are several approaches to local class field theory: 1) Hasse's approach (1930s) via the theory of central simple algebras and Brauer groups; 2) the Galois-cohomological approach developed in the 1950s by Artin, Hochschild, Nakayama, Tate and Weil; 3) the formal group approach, developed in the mid 1960s by Lubin and Tate; 4) the more modern and more direct approaches of Hazewinkel (1970s) and Neukirch (1980s).

Global class field theory is the precise description of the Galois group of the maximal abelian extension of a **global field.** It is the crowning achievement of work performed during the late 1800s and the first half of the 20^{th} century by several outstanding mathematicians such as Kronecker, Weber, Hilbert, Takagi, Artin, Hasse, Chevalley, Tate, Nakayama,Weil.

In the Spring 2006 (Math 205C), we covered in detail the main properties of local fields and their finite extensions. Also, we stated (without complete proofs, but with plenty of examples) the main theorem of local class-field theory – **the local reciprocity law,** and gave a fairly detailed Galois-cohomological description of **the local reciprocity map.**

During the first two weeks of Fall 2006, we will review (without proofs but with plenty of examples) the material covered last Spring. This will be followed by a detailed description of the Lubin-Tate explicit construction of the maximal abelian extension of an arbitrary local field via torsion points of **Lubin-Tate formal groups,** as well as the complete characterization of the local reciprocity map in terms of these torsion points. Then, we will move on to the study of global class field theory. This will entail a thorough study of the **idèle group and the various idèle-class groups** associated to an arbitrary global field and the Chevalley-Artin-Tate description of the global reciprocity map linking the Galois group of the maximal abelian extension of this global field to its idèle-group. Time permitting, we might sketch the more modern Galois-cohomological approach to global class field theory due to Artin and Tate as well. Finally, given an arbitray global field, we will establish a link between its global reciprocity map and the local reciprocity maps associated to its completions with respect to all its valuations (metrics).

The recommended bibliography for this course is as follows.

Local Class Field Theory

1. *K. Iwasawa,* **Local Class Field Theory,** Oxford University Press (1986)
2. *J.P. Serre,* **Local Fields,** Second Edition, Springer Verlag (1995)
3. *I. Fesenko, S. Vostokov,* **Local Fields and Their Extensions,** Second Edition, AMS Translations of Math. Monographs (2001)

Global Class Field Theory

1. *E. Artin and J. Tate,* **Class Field Theory,** *Benjamin (1967)*
2. *J. Neukirch,* **Algebraic Number Theory,** *Springer Verlag (1999).*
3. *S. Lang,* **Algebraic Number Theory,** *Addison-Wesley (1970)*

Graduate and advanced undergraduate students are invited to attend. Knowledge of graduate level abstract algebra and basic algebraic number theory (e.g. global fields and their basic properties) will be helpful.

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